Worksheet #18: Applications of the Integral

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Graph the following function using [xMin,xMax] = [-8,8]:

$$N(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

This is a bell shape curve. Think of this graph as a histogram. When doing research and testing, data often falls into this shape and if it does, we say the data has a normal distribution. There are two parameters associated with a normal distribution: mean value (μ) and the variance (σ^2). We use the notation $N(\mu, \sigma^2)$ to describe a normal distribution with mean μ and variance σ^2. Note that the standard derivation of the data will be σ if the variance is σ^2. The function above describes the shape of a normal distribution $N(0, 1)$. $N(75, 25)$ would describe data that has mean 75 and a standard deviation of 5.

Compute the following integrals.

$$\int_{-8}^{8} N(x)dx \qquad \int_{-8}^{-6} N(x)dx \qquad \int_{6}^{8} N(x)dx \qquad \int_{-8}^{0} N(x)dx$$

The integral of $N(x)$ can be intepreted as a probability. The probability that X is less than α can be written

$$P(X < \alpha) = \int_{-\infty}^{\alpha} N(x)dx$$

So, the probability that X is less than 2 will be $\int_{-\infty}^{2} N(x)dx$. What do the four integral above tell us?

Let T be a random variable (perhaps a test score) and you want to know what is the probability that the test score T is less than 80. Let's assume we have test scores that fit a normal distribution of $N(75, 25)$. We have to convert our application to a normal distribution of $N(0, 1)$ to compute the probability using $N(x)$.

$$P(T < 80) = P(T - 75 < 5) = P(\frac{T - 75}{5} < 1)$$

$$P(X < 1) = \int_{-\infty}^{1} N(x)dx$$

By subtracting the mean and dividing by the standard deviation, we convert $N(75, 25)$ data into $N(0, 1)$ data. Thus we can use the integral to compute the probability.

- Given $N(0, 1)$, find probability that $X < 2, X < -2$, and $X > 2$
- Given $N(0, 1)$, find probability that $-2 < X < 2$
- Given $N(75, 25)$, find probability that $T < 90, T < 65$, and $T > 65$
- Given $N(75, 25)$, find probability that $65 < T < 85$