

Worksheet #17: Applications of the Integral

April 15, 2005

#6. A forest fire covers 2000 acres at time $t=0$. The fire is growing at a rate of $R = 8\sqrt{t}$ acres per hour, where t is in hours. How many acres are covered 24 hours later?

The integral of the rate will give you the number of acres covered in whatever timeframe you select. $\int_0^{24} R dt$ will give you the number of acres covered between $t = 0$ and $t = 24$. Add this number to 2000 and you get the total coverage.

#9. The amount of waste a company produces, W , in metric tons per week, is approximated by $W = 3.75e^{-0.008t}$, where t is in weeks since January 1, 2000. Waste removal for the company costs \$15/ton. How much does the company pay for waste removal during the year 2000.

The integral of the rate at which waste is produced will tell you how much waste was produced during that timeframe. The area under the curve is equivalent to the number of tons of waste that were produced. How many weeks are in a year? Now, once you have the number of tons of waste, you get the cost of removal by multiplying by \$15/ton.

#11. Salesman Problem.

The integral of the number of sales per week will tell you the number of sales during that timeframe. What does the area between the two curves represent? It is obvious that Salesman A is doing better at first, but the area between the curves tells you exactly how much? At some point, their rates are the same. Who has sold more at this point?

#12. After a foreign substance is introduced into the blood, the rate at which antibodies are made is given by:

$$r(t) = \frac{t}{t^2 + 1}$$

thousands of antibodies per minute. How many antibodies are there after 4 minutes?

The integral of $r(t)$ (because it is a rate) is the number of antibodies produced during that timeframe. The area under the curve is equivalent to the number of antibodies. So, $\int_0^4 r(t) dt$ will tell you how many were produced in the first 4 minutes. Since there were no antibodies at $t = 0$, then this is also the number in the bloodstream when $t = 4$.

#16. The velocity of a car (in mph) is given by $v(t) = 40t - 10t^2$, where t is in hours.

This is just like the other worksheets. The integral of the velocity tells you the distance traveled. Area under the curve is equivalent to the distance traveled.