Worksheet #8 - Instantaneous Rate of Change (Solutions)

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Please print your name(s) at the top of the page. Please answer in complete sentence.

Graph the following function on your calculator using the domain of $x \in [2, 11]$

$$y = \cos(-x^2/8) \cdot (x - 6)$$

- Determine the instantaneous rate of change of the function at $x = 4$ by using $\Delta x = 1, 0.1, 0.001$.

For each Δx, you will have to compute a different instantaneous rate of change (actually, you are approximating the instantaneous rate of change). So using, $\Delta x = 1$,

$$f'(4) \approx \frac{f(5) - f(4)}{5 - 4} = \frac{.99986 - .83229}{1} = .16757$$

Again, only an approximation. So using, $\Delta x = 0.1$,

$$f'(4) \approx \frac{f(4.1) - f(4)}{4.1 - 4} = \frac{.96126 - .83229}{0.1} = 1.2897$$

Again, only an approximation, but using smaller values for Δx, you will get a better approximation. So using, $\Delta x = 0.001$,

$$f'(4) \approx \frac{f(4.001) - f(4)}{4.001 - 4} = \frac{.83369 - .83229}{0.001} = 1.4$$

Notice that you should use more decimal places in your calculations as you make Δx smaller.

- Determine the instantaneous rate of change of the function at $x = 8.5$ by using $\Delta x = 2, 0.2, 0.002$.

Use the slope formula with $x = 8.5$ and the different values of Δx. For example,

$$f'(8.5) \approx \frac{f(8.502) - f(8.5)}{0.002} = \frac{(-2.314809) - (-2.308905)}{0.002} = -2.952$$

Notice that the point $(8.5, -2.308905)$ is located at a place where the function is decreasing. The derivative (or instantaneous rate of change) at $x = 8.5$ is negative, which implies the function is decreasing.

- Investigate the "Draw-Tangent" feature on your calculator. Draw a tangent line on the graph at $x = 4$ and another at $x = 8.5$. How can this feature help you in answer the questions above?

  After drawing the tangent line, the equation of the line is given at the bottom of the screen. The slope of this line is same as the instantaneous rate of change.

- Using your eyes, find the approximate locations where the instantaneous rate of change is equal to zero? What are the terms we have used for these type of points? With respect to the shape of the function, how are they different?

  The locations where the instantaneous rate of change is equal to zero are the peaks and valley of the graph. Earlier in the semester, we used terms like "critical points" or "local minima" or "local maxima" to describe these points. Notice that the local minima occur when the graph is concave up and the derivative is equal to zero. The local maxima occurs when the graph is concave down and the derivative is equal to zero.

- Name a function has the instantaneous rate of change always equal to 2? Plot this function. Using your eyes, find the approximate locations where the instantaneous rate of change is equal to 2?

  The line $y = 2x$ has a constant instantaneous rate of change of 2. Places where the function $f'(x)$ will equal 2 will be places where the tangent lines to the graphs will be parallel to the line defined by $y = 2x$.