Worksheet #22: Application of the Integral

April 27, 2006

Here are two problems that you can work on to practice for the upcoming final. I've included solutions.

(Moe's Problem) I estimate that I go to Moe's approximately 5 times every two weeks. I went there today for lunch. What is the probability that I will hear "Welcome to Moe's" tomorrow at lunch time ?

First, this is an exponential distribution problem. You have to find the waiting time parameter μ in order to have the correct function that you will eventually use within the integral. So, to eat 5 burritos every two weeks (or 14 days) means I wait $\frac{14}{5}$ days between burritos. This means $\mu = \frac{14}{5}$.

Remember in these problems the probability will equal the integral of $\frac{1}{\mu}e^{-w/\mu}$ where the question will provide you the lower and upper integrands. The question asked for the probability that I will go to Moe's the next day - in other words, the probability that I will wait a day or less to have another burrito.

$$P(W < 1) = \int_0^1 \frac{5}{14}e^{-5w/14} \quad dw$$

Graph $y = \frac{5}{14}e^{-5w/14}$ and use Calc - Option 7 to determine the probability. The integral turns out to be 0.3003, so I have a 30.03% chance of going to Moe's on back-to-back days.

Do you know how to setup the integral that tells you the probability that I will have to wait more than 3 days to have a burrito? There was a little issue when infinity is your upper integrand - do you know the trick to avoid having to use infinity. Hint:

$$P(W \leq 3) + P(W \geq 3) = 1$$

(Length of a Curve) What is the length of the portion of the parabola $y = 14 + 5x - x^2$ that is above the x-axis ?

The formula to find the length of the curve is

$$S = \int_a^b \sqrt{1 + [f'(x)]^2} \quad dx$$

where in this problem, $f(x) = 14 + 5x - x^2$. By factoring, you would see that $14 + 5x - x^2 = (7 - x)(2 + x)$. This tells us that the parabola has roots at $x = 7$ and $x = -2$, and since the coefficient of the x^2 term is -1, we know that the parabola is concave down, hence the portion of

the graph between $x = -2$ and $x = 7$ is above the x-axis. So, $a = -2$ and $b = 7$. The final answer would be

$$S = \int_{-2}^{7} \sqrt{1 + [5 - 2x]^2} \quad dx$$

At this point, you have to graph $\sqrt{1 + [5 - 2x]^2}$ and find the area under this graph between $x = -2$ and $x = 7$. The final answer is 42.196. To avoid algebra mistakes, I would advise not simplifying the expression under the square root symbol, just type it in properly into your calculator.

Do you know how the formula for S was derived? It is based on the Pythagorean Theorem that you've been studying since middle school.