Problems and Solutions - Problem 33, page 51

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Here is a writeup for Problem 33 from page 51. This is a problem where you have to compare two plans that are giving you money and determine which is a better plan. The key is to convert all of the money (today's money, money given a year in the future, money given 10 years in the future) to the same time reference. To convert from future value to present value, you use the following formula:

$$P = \frac{F}{(1+r)^t}$$

where F is the future value, r is the annual rate, and t is number of years that the money grows.

> *Problem: Interest is compounded annually at the rate of 5% per year. Consider the following choices of payments to you.*
>
> - *Choice #1: \$1500 now and \$3000 one year from now*
> - *Choice #2: \$1900 now and \$2500 one year from now*

First, I find building a chart to be the best way of organizing these types of problems. Make a chart listing the different years for your column headings ($t = 0$ and $t = 1$, in this problem) and then enter the known values under the appropriate year. So, under $t = 0$, you will have an entry for \$1500 and \$1900, while under $t = 1$, you will have an entry for \$3000 and \$2500.

Present Value	$t = 0$	$t = 1$
	1500	
		3000
	1900	
		2500

Now, determine the present value of each of these dollar amounts. For $t = 0$, these are easy, since $t = 0$ means the present in these problems. But for $t = 1$, we have to use the formulas. For the first plan, the present value of \$3000 is:

$$P = \frac{3000}{1.05} = 2857.14$$

For the second plan, the present value of \$2500 is:

$$P = \frac{2500}{1.05} = 2380.95$$

Now filling the chart.

Present Value	$t = 0$	$t = 1$
1500	1500	
2857.14		3000
1900	1900	
2380.95		2500

So, the first choice has net present value of \$4357.14, while the second plan has a net present value of \$4280.95. So, it is in your best for you to choose the first plan, since it has more value than the second.

The second question of this problem takes a little bit a thinking, but you have the math skills to answer it. So, the question is: Is there an interest rate that would lead you to make a different choice? Let's play with some numbers to answer this question. Then we will do a little algebra to solve a more specific question - that is, at what percentage rate are both plans considered to be equal?

First, let's setup the functions that describe how much money each plan is worth as a function of r (the rate at which the money grows). Let C_1 be the net present value of Choice #1 and let C_2 be the net present value of Choice #2. These formulas are obtained by doing the same steps that we did to build the chart above, but using r (not 5%) for the rate since it is now our independent variable.

$$C_1 = 1500 + \frac{3000}{1+r}$$

$$C_2 = 1900 + \frac{2500}{1+r}$$

You should plot these functions on your calculator. Let your x-values range from 0 to 1, while your y-values will range between 3000 and 5000.

Let's look at the extreme cases. What if we earned nothing on our money? This would be where $r = 0$. This would mean if we had \$100 today, we'd have \$100 in a year, and the same \$100 in 10 years. So, in this case, Choice #1 is worth \$4500 today and Choice #2 is worth \$4400. For $r = 0$ (no growth in your money), Choice #1 would be a better deal.

The other extreme case would be if you were able to earn a very high rate of return on your money. Notice that if r is a very large number, the fractions $\frac{3000}{1+r}$ and $\frac{2500}{1+r}$ would be very small numbers and would not contribute much to net values of C_1 or C_2. If you were going to receive a large interest rate, you'd like to have as much money up front as possible. So, for this case, you want to have the \$1900 of Choice #2 rather than only \$1500 in the other plan. The money that you get in the second year does not play a role determining which method is better - in this case where r is very large, Choice #2 is better.

But the next question is when are the plans the same. At what value of r does it not make a difference if you pick the first or second plan? If you have plotted the functions, then the answer is where the two curves cross each other. Algebraically, if you set the two functions equal to each other, then solve you get the following solution.

$$1500 + \frac{3000}{1+r} = 1900 + \frac{2500}{1+r}$$

$$\frac{3000}{1+r} - \frac{2500}{1+r} = 1900 - 1500$$

$$\frac{500}{1+r} = 400$$

$$1 + r = \frac{500}{400}$$
$$r = 1.25 - 1 = 0.25$$

Hence, at the rate of 25%, both plans are equal.

This is a very good problem. Here are the key points that you should get out of this problem.

- How to determine the net value of any individual plan. This is done through building the table and computing the present value of money.
- How to compare two plans given the net present value of each plan? In this problem, the larger net value was better. There was a problem about servicing your dishwasher where the lower net value was the better value, since you were spending money.
- In the second part of this problem, understanding how the interest rate controls the net present value of the plans.
- Most importantly, one of your classmates asked me about this problem and I couldn't answer it on the spot. For me, it required writing out the formulas and plugging in some numbers to get a feel of what was happening. Also, after the formulas for C_1 and C_2 were obtained, seeing the graphs of these functions (using the calculator) was very helpful. Bottom line is that some of these problems can not be answer immediately - a little effort is required.