HW Test #8 - Limits (Solution)

March 1, 2006

By replacing the question marks (???) with the correct notation, re-write the following definition of the derivative of $f(x)$ at $x = 5$.

$$f'(5) = \lim_{???} \frac{???}{h}$$

The definition of a derivative is basically a slope formula. The numerator will be the difference of two function values, while the denominator (and it is given to be h) is the difference of two x values. The two points that are used to compute the slope have two properties: 1) they are both on the graph and 2) they are very close to each other. So, we pick $(5, f(5))$ and $(5+h, f(5+h))$ - both are on the graph and when $h \to 0$, we force them to be close to each other.
So, the final answer is:

$$f'(5) = \lim_{h \to 0} \frac{f(5+h) - f(5)}{h}$$

Now, to approximate this limit, we set h to be very small, $h = 0.0001$ and compute:

$$f'(5) \approx \frac{f(5.0001) - f(5)}{0.0001}$$

Evaluate each of the limits. Do not base your answer on simply one computation - use enough estimates until they match within three decimal places. Present your work in an orderly fashion.

$$\lim_{n \to \infty} \frac{8n^3 - 64}{5n^3 - 2}$$

The expression above answers the following question: What is the value of $\frac{8n^3-64}{5n^3-2}$ when n gets very, very large? So, you would need to plug in values of n that are very large (start with $n = 100$, then $n = 10000$, and so on). Once you recognize that the numbers are approaching something, then you can stop - this is when the numbers match up to several decimal places.
If you plug in $n = 100$, $n = 1000$, and $n = 10000$, you will recognize that the values will be very close to 1.6.

$$\lim_{n\to\infty}\left(1+\frac{4}{n}\right)^n$$

The expression above answers the following question: What is the value of $\left(1+\frac{4}{n}\right)^n$ *when n gets very, very large? So, you would need to plug in values of n that are very large (start with* $n = 100$*, then* $n = 1000$*, and so on). Once you recognize that the numbers are approaching something, then you can stop - this is when the numbers match up to several decimal places.*

If you plug in $n = 100$*,* $n = 1000$*, and* $n = 10000$*, you will get values of 50.504, 54.164, and 54.5545. This look like they are going to approach some number between 54 and 55, but the decimal places haven't match up yet. You need to plug in larger n values. Let* $n = 1,000,000$ *and* $n = 2,000,000$*, and you will get 54.5977 and 54.5979. So, you see the numbers are finally matching. The final answer would be that 54.598 (rounding up the last digit).*

Some people wrote 55. Not all answers are integers. If the number was 2.99999, then I would round up to 3, but the values in this problem were not approaching 55, they were approaching 54.598.