Worksheet #15 - Average Value

Spring 2007

Objectives

- Learning a new application of the integral
- Determining the average value of a function over an interval

Background You have taken the average of numbers before. This did not require calculus. For practice, take the average of the following numbers. Consider the process that it takes to take the average of a finite number of values.

$$30, 28, 25, 22, 18, 15, 19, 22, 23, 24$$

The process is to sum the values and divided by the cardinality of the set. In this case, the average is 22.6 (226/10).

Question How would you compute the average speed of your car over a certain length of time? The speed of your car is a function of time and is not discrete. But you could write down the car's speed every 10 minutes or every 30 seconds and take the average of the data that you collected.

Average Value of a Function Let's consider the more general case. Suppose you had a function $f(t)$ and wanted to know the average value of the function between $t = a$ and $t = b$. Let's pick N times to measure the function and take the average of these values.

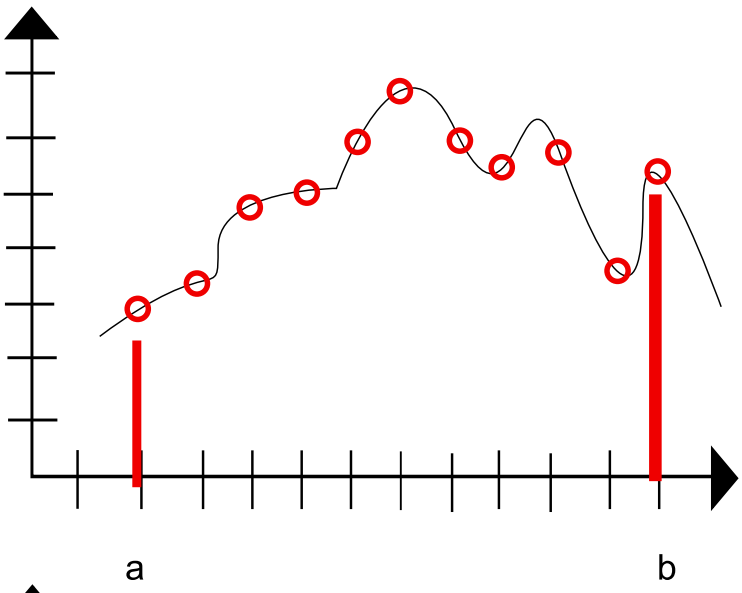
$$\begin{gathered} a = t_0, \quad t_1, \quad t_2, \quad \ldots \quad , \quad t_{n-1}, \quad t_n = b \\ y_0 = f(t_0), \quad y_1 = f(t_1), \quad \ldots \quad , \quad y_N = f(t_N) \end{gathered}$$

$$\text{Average Value } = \frac{1}{N} \sum y_i \quad = \frac{1}{N} \sum f(t_i)$$

$$\begin{gathered} \Delta t = \frac{b-a}{N} \\ \frac{1}{N} = \frac{\Delta t}{b-a} \end{gathered}$$

$$\begin{aligned} \text{Average Value } &= \frac{\Delta t}{b-a} \sum f(t_i) = \frac{1}{b-a} \sum f(t_i) \Delta t \\ &\approx \frac{1}{b-a} \text{ (Area under the Curve)} \\ &= \frac{1}{b-a} \int f(t) dt \end{aligned}$$

Average Value $\cdot (b-a) =$ Area under the Curve



Area of Rectange = Area under Curve