Worksheet #23 - Present and Future Value

Spring 2007

Objectives

- How to compute the value of today's money in the future
- How to compute future money in today's dollars
- Applications include pricing a bond, a service contract, or a stock investment

Present and Future Value

- Assumption is that money and investment grow exponentially.
- The growth rate of a long-term (30 year) Treasury bond is used as a benchmark since these bonds are low-risk.
- Formulas for Future Value (when given annual or continuous rates)

$$F = P(1+r)^t \qquad\qquad F = P \cdot e^{rt}$$

- Formulas for Present Value (when given annual or continuous rates)

$$P = F/(1+r)^t \qquad\qquad P = F \cdot e^{-rt}$$

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Fill in the charts with either the future or present values assuming a 5% annual interest rate.

$r = 5\%$	1 year	2 years	3 years	4 years
\$200				
				\$4000

Use the $F = P(1 + r)^t$ formula to advance \$200 to the proper year or just multiply the previous year by $(1 + r)$ to make the money grow one year at a time. Use $P = F(1 + r)^{-t}$ to backtrack the \$4000 dollars.

$r = 5\%$	1 year	2 years	3 years	4 years
\$200				$200(1.05)^4$
$4000(1.05)^4$				\$4000

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Do you have enough money in the bank today to pay for future purchases if you could make your money grow at a rate of 10% continuously?

$r = 10\%$	1 year	2 years	3 years	4 years
\$2000				
	500	500	1000	500

Backtrack all the future expenses into today's (present) value. The sum of these present values is equivalent to the value of the future expenses. If you have more money in the bank than the sum of the present values, then you have enough to buy the items in the future when the time comes.

$r = 10\%$	1 year	2 years	3 years	4 years
\$2000				
$500 \cdot e^{-0.10} = 452.42$	500			
$500 \cdot e^{-0.20} = 409.36$		500		
$500 \cdot e^{-0.30} = 740.82$			1000	
$500 \cdot e^{-0.40} = 335.16$				500
Total = 1937.76	500	500	1000	500

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Find out the present value of these payments. This is a type of bond payment. How much would you pay for this bond? Assume this is an annual rate.

$r = 10\%$	1 year	2 years	3 years	4 years
	100			
		100		
			100	
				1000 + 100

Backtrack all the future payments into present value dollars. The sum of the present values is equivalent to the value of the future payments. It is the most you should be willing to pay if you could invest your money and earn 10% each year. After seeing the answer, does it make sense why this should be the answer?

$r = 10\%$	1 year	2 years	3 years	4 years
90.90	100			
82.64		100		
75.13			100	
751.31				1000 + 100
Make Sense = 1000.00	100	100	100	1000 + 100

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What would be the loan payment that you would receive if you lend \$2000 at a fair interest rate (8% annual rate) for four years? For each payment, compute the present value of L dollars. These numbers should sum to \$2000.

$r = 8\%$	1 year	2 years	3 years	4 years
	L			
		L		
			L	
				L
2000				

Backtrack each of the payment amounts L. Since you don't know L going into the problem, you have to use variables and use formulas that would tell you the present value of each of the future payments. With all the present values of the payments, sum the present values and set them equal to the loan amount. You can now factor out the L and divide to get the value of L.

$$\frac{L}{1.08} + \frac{L}{1.08^2} + \frac{L}{1.08^3} + \frac{L}{1.08^4} = 2000$$

$$L\left\{\frac{1}{1.08} + \frac{1}{1.08^2} + \frac{1}{1.08^3} + \frac{1}{1.08^4}\right\} = 2000$$

$$L\left\{3.3121\right\} = 2000$$

$$L = \frac{2000}{3.3121} = 603.84$$

So, payments of \$603.84 over four years is equivalent to \$2000 today.