Worksheet #20: Exponential Distribution

Spring 2007

Objectives

- Learning the integral feature of your calculator
- Relating area under a curve to probability
- Introduce the exponential probability density function
- Answer questions related to waiting times

Background information.

- Exponential distribution use the following probability density function.

$$E(\omega) = \frac{1}{\lambda} \cdot e^{-\omega/\lambda}$$

- $E(\omega)$ is a exponential decay function. The parameter λ is the average waiting time - remember it is usually given in units of seconds, minutes, hours, etc.
- If you work at a grocery store and observe that you have on average 20 customers per hour, then you see on average 1 customer every 3 minutes. Your average waiting time for a customer is 3 minutes. In this case, you would use $\lambda = 3$ in the function for $E(\omega)$.
- Graph $E(\omega)$ with $\lambda = 3$. Use $[xMin, xMax] = [0, 15]$. In the future problems, use $[xMin, xMax] = [0, 5\lambda]$. Compute the area under the graph from $x = 0$ to $x = 15$.
- Once you have the waiting time, one question that you might ask yourself is "What is the probability that the store will have a new customer in the next 2 minutes?" or "What is the probability that the store will not have a new customer in the next 5 minutes?"

$$P(W \leq 2) \qquad\qquad P(W \geq 5)$$

- To answer these questions, we have to compute the area under $E(\omega)$ function with the proper lower and upper limits. Remember that to compute the area under the curve, you must graph the function $E(\omega)$ on the display and use Calc-Option #7. This option will require you to input the lower and upper limits.
- Here is how we relate the answers to the probability questions to the area under the curve.

$$P(W \leq 2) = P(0 \leq W \leq 2) = \int_0^2 E(\omega) \cdot d\omega = 0.4866 = 48.66\%$$

$$P(W \geq 5) = 1 - P(W \leq 5) = 1 - \int_0^5 E(\omega) \cdot d\omega = 1 - 0.8111 = 18.89\%$$

- In the end, you can make a statement from your calculations:

  There is a 48.66% chance that the store will have a new customer in the next 2 minutes.

  There is a 18.89% chance that the store will not have a new customer in the next 5 minutes.

Problems: Compute λ (average wait time).

- In the grocery store situation, what would the average wait time be if you saw 15 customers per hour?

- In the grocery store situation, what would the average wait time be if you saw 25 customers per hour?

- While working at a help desk, you measure the time between calls. Compute the average wait time if you receive two calls every five minutes.

- While work for the DOT, your job is to measure the time between blue cars that pass a location on the highway. What is the average wait time if you see 3 blue cars every minute?

- During the summer, you work in the quality control department of a candy factory and measure the time between flaws on an assembly line. You detecting 10 flawed pieces of candy for every 800 pieces that are produced. What is the average wait time between flawed pieces of candy? In this case, time isn't measured in the usual units.

Problems: Compute the average wait time parameter, graph the proper probability density function, set up the proper integral expression that is equivalent to the probability being asked, and answer the probability question using the integral feature of your calculator.

- If the help desk handles 6 calls per hour, what is the probability that you will have to wait 30 minutes for a call?

- If the help desk handles 6 calls per hour, what is the probability that you will receive a call within 5 minutes of the last call?

- If the help desk handles 12 calls per hour, what is the probability that you will receive a call within 5 minutes of the last call? Compare this answer to the help desk that handles 6 calls per hour.

- If the help desk handles 12 calls per hour, what is the probability that you will receive a call between 5 and 10 minutes of the last call?

- If you work at a candy factory and see 8 flawed candy for every 1000 pieces that are produced, what is the probability that you will see a flawed piece of candy in the next 50 pieces that are being produced?

- If you work at a candy factory and see 8 flawed candy for every 1000 pieces that are produced, what is the probability that you will not see a flawed piece of candy in the next 200 pieces that are being produced?