Worksheet #2 - Basic Algebra

Spring 2007

Objectives

- Review some of basic algebra rules.
- Students make more algebra mistakes in this class than calculus mistakes.
- Know what you are allowed and are not allowed to do when given an expression or an equation.
- Think of more than one way to solve or answer these questions.

Problems

- Solve for x when $30x - 100 = 20x + 40$.

  One way is to move the constant terms to one side of the equality and move the variables to the others. Then, divide by the coefficient. This works for equations with one variable term.

  $$30x - 100 = 20x + 40$$

  $$10x = 140$$

  $$x = 14$$

  Double check your answer: $30 \cdot 14 - 100 = 420 - 100 = 320$

  Same answer for the right side: $20 \cdot 14 + 40 = 280 + 40 = 320$

- Solve for t when $4t^2 - 10t - 20 = 4t^2 + 40t + 120$.

  One way to solve this would be to simplify the expression. The $4t^2$ *cancels out on both sides, leaving* $-10t - 20 = 40t + 120$. *Now apply the previous strategy,*

  $$-10t - 20 = 40t + 120$$

  $$-50t = 140$$

  $$t = -\frac{140}{50} = -\frac{14}{5} = -2.8$$

  Let's think about how we could use our calculators.
  Another way to solve this using your calculator would be to graph

the two functions $y = 4t^2 - 10t - 20$ and $y = 4t^2 + 40t + 120$, then the solution would be the first coordinate of the point of intersection. Using your calculator may give you an approximation to the true solution, but that's ok. This is a good method to use to solve any problem where two functions equal each other. Again, perhaps only an approximation to the true answer but at least it is a method that will produce something (close).

- Solve for t when $t^2 - 10t - 20 = 0$.

  This is quadratic so there are several ways that this can be solved. First, quadratic formula would do the trick. Other way would be completing the square; let's go over that.

$$t^2 - 10t - 20 = 0$$
$$t^2 - 10t + \_\_\_ = 20$$
$$t^2 - 10t + 25 = 20 + 25$$
$$(t-5)^2 = 45$$
$$t - 5 = \pm\sqrt{45} = \pm 3\sqrt{5}$$
$$t = 5 \pm 3\sqrt{5}$$

  Yet, another way is to use your calculator to graph $y = t^2 - 10t - 20$ and location the places where $y = 0$ which would be the places where the graph crosses the t-axis. Using your calculator may give you an approximation to the true solution, but that's ok.

- Solve $A = \sqrt{49 + 16}$. Graph $y = \sqrt{t^2 + 16}$. What is y when $t = 7$.

  The main point in this problem is to show that you can not simplify $\sqrt{49 + 16} = \sqrt{65}$ as $\sqrt{49} + \sqrt{16} = 7 + 4$. So, do not make the classic mistakes of thinking that $\sqrt{65} = 11$.

- Factor the following expression: $B = 5t^4 - 20t^3$. What is B when $t = 2$?

  When factoring, we are trying to represent the orginal expression as the product of two simplier expressions. Notice that both terms of the expression share a t^3 and 5 in common. Therefore, $5t^3$ can be factored out. both

$$B = 5t^4 - 20t^3 = 5t^3 \cdot (t - 4)$$

Now, determining B when $t = 2$ can be found using either form of B. So,

$$B(2) = 5 \cdot 2^3 \cdot (2 - 4) = 5 \cdot 8 \cdot (-2) = -80$$

- Factor $x^2 - 30x + 200$.

  This factoring requires making guesses based on the -30 and 200 in the quadratic. You need to find two numbers whose product is 200 and whose sum is 30. Sometimes, you won't be able to guess, because the numbers will be irrational or perhaps even complex. But in this case, you may find that 10 and 20 are two numbers that work for our problem.

  $$x^2 - 30x + 200 = (x - 10)(x - 20)$$

- Simplify $\frac{x^4 - 40x^2}{x^5 + x^2}$. How can you check your answer?

  To simplify the expression, we need to find factors that the numerator and demoninator have in common. Notice that all the terms have x^2 in common.

  $$\frac{x^4 - 40x^2}{x^5 + x^2} = \frac{x^2(x^2 - 40)}{x^2(x^3 + 1)} = \frac{x^2 - 40}{x^3 + 1}$$

  There is a chance that $x^2 - 40$ and $x^3 + 1$ share a common factor, but they don't.

  $x^2 - 40 = (x - \sqrt{40})(x + \sqrt{40})$ $\qquad$ $x^3 + 1 = (x + 1)(x^2 - x + 1)$

- Simplify $(2x + 3)(4x + 1)$. How can you check your answer?

  OK, maybe the problem should not have said simplify, but expand the expression. You can use the FOIL method.

  $$(2x + 3)(4x + 1) = 8x^2 + 2x + 12x + 3 = 8x^2 + 14x + 3$$

  You can check you answer to by putting in some different values of x. Like for $x = 0$, you should get $3 \cdot 1$ and if you plug into the expanded versions, you get 3. Good. Try one more number. For $x = 1$, you get $5 \cdot 5$ versus $8 + 14 + 3$. Both are 25. Good.

- Simplify $A = [8e^{3t}][10e^{4t}][2e^{-5t}]$

Multiplying exponentials with a similar base, then you just add the exponents.

$$A = [8e^{3t}][10e^{4t}][2e^{-5t}] = 160e^{3t+4t-5t} = 160e^{2t}$$

- Find the value of S when $t = 2$ if $S = 200te^{-4t+5}$. Paratheses might be necessary when using your calculator.

  Here is what you can type to get the correct answer.

`200 * 2 * e^(-4*2+5)  but not 200 * 2 * e^-4*2+5`