# Worksheet #18: More Probability

Spring 2007

### Objectives

- Learning the integral feature of your calculator
- Relating area under a curve to probability
- Introduce the normal probability density function
- Answer questions related to data fitting a bell shaped curve

Graph the following function using [xMin,xMax] = [-7,7]. Change the [yMin,yMax] so the curve fills your calculator screen.

$$N(x) = \frac{e^{-x^2/2}}{\sqrt{2\pi}}$$

**Normal Distributions.** This is a bell shape curve. Think of this graph as a histogram. When doing research and testing, data often falls into this shape and if it does, we say the data has a normal distribution. There are two parameters associated with a normal distribution: mean value ($\mu$) and the variance ($\sigma^2$). We use the notation $N(\mu, \sigma^2)$ to describe a normal distribution with mean $\mu$ and variance $\sigma^2$. Note that the standard derivation of the data will be $\sigma$ if the variance is $\sigma^2$. The function above describes the shape of a normal distribution $N(0, 1)$.

**Integrals and Area under the Curve.** The expression below can be read as "the integral of $N(x)$ between $x = -5$ and $x = 5$" and represents the area under the graph between the $x$-values.

$$\int_{-5}^{5} N(x)dx$$

Your calculators can evaluate the expression when given the right inputs.

- First, you have to graph the function $N(x)$.
- Then, use the Calc-Option 7 feature.
- You will use -5 for the lower limit and use 5 for the upper limit.
- The answer is that appears at the bottom of your display is the area.

**Calculator Exercises.** Use your calculator to answer the following questions.

- What is approximately the area under this curve between $x = -5$ and $x = 5$?
- You can clear the graph by typing 2nd-Draw-Option 1.
- What is the area (use 4 decimal places) between $x = -1$ and $x = 1$?
- What is the area (use 4 decimal places) between $x = 0$ and $x = 2$?

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**Area as Probability.** The integral of $N(x)$ can be interpreted as a probability. Notice that all the areas were positive and less than or equal to 1. Remember the histogram that were created for the rolling of two dice - the bars were equivalent to the chances that a roll would occur.

The probability that $X$ is less than $\alpha$ can be written

$$P(X < \alpha) = \int_{-\infty}^{\alpha} N(x)dx$$

So, the probability that something that you are measuring ($X$) is less than 2 will be $\int_{-\infty}^{2} N(x)dx$. For these problems, we will use -5 for $-\infty$ when using the calculators.

The probability that $X$ is greater than $\alpha$ and less than $\beta$ can be written

$$P(\alpha < X < \beta) = \int_{\alpha}^{\beta} N(x)dx$$

- What are the chances that a number that is randomly pick will be less than zero?
- What are the chances that a number that is randomly pick will be less than 1?
- What are the chances that a number that is randomly pick will fall between $x = -1$ and $x = 1$?
- Don't forget to convert all the areas into percentages.

**Other Normal Distributions Functions.** Sometimes, data still has a normal distribution, but is not of a $N(0, 1))$ distribution. Let T be a random variable (perhaps a test score) and you want to know what is the probability that the test score T is less than 80. Let's assume we have test scores that fit a normal distribution of $N(75, 25)$. $N(75, 25)$ would describe data that has mean 75 and a standard deviation of 5.

We might ask questions like:

- What are the chances a test score falls between 70 and 80?
- What are the chances a test score will be below a 72 ?

To answer these questions, we will convert the question so that it is asked using a N(0,1) distribution. The conversion requires subtracting the mean and dividing by the standard deviation. Then you take the integral to find the probability.

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(Repeated) To answer these questions, we will convert the question so that it is asked using a N(0,1) distribution. The conversion requires subtracting the mean and dividing by the standard deviation. Then you take the integral to find the probability.

- What are the chances a test score falls between 70 and 80?

$$70 < T < 80$$

$$-5 < T - 75 < 5$$

$$-1 < \frac{T-75}{5} < 1$$

$$P(70 < T < 80) = P(-1 < X < 1) = \int_{-1}^{1} N(x)dx$$

- What are the chances a test score will be below a 72 ?

$$T < 72$$

$$T - 75 < -3$$

$$\frac{T-75}{5} < -0.6$$

$$P(T < 72) = P(X < -0.6) = \int_{-\infty}^{-0.6} N(x)dx$$

**Homework.** For each of these problems, do the following steps: 1) determine the mean, variance, and standard deviation of the data, 2) draw a bell shape curve, label the mean and one standard derivation away from the mean, 3) shade the region that corresponds to the probability that the event will happen, and 4) compute the probability. 5) write a statement about the event and its probability of happening.

- Given $N(0,1)$, find probability that $-1 < X < 1$

$$P(-1 < X < 1) = \int_{-1}^{1} N(x)dx = 68.27\%$$

*This is the probability that the event will fall within one standard deviation of the mean.*

- Given $N(0,1)$, find probability that $X < 0.5$

$$P(X < 0.5) = \int_{-\infty}^{0.5} N(x)dx = 19.15\%$$

- Given $N(0,1)$, find probability that $X > 1.2$

$$P(X > 1.2) = 1 - \int_{-\infty}^{1.2} N(x)dx = 1 - 88.49\% = 11.51$$

- (Test Scores) Given $N(80,25)$, find probability that $77 < T < 81$

*You have to transform the problem from using the variable $T$ of the $N(80,25)$ distribution into the variable $X$ of the $N(0,1)$ distribution.*

$$77 < T < 81$$
$$-3 < T - \mu < 1$$
$$-0.6 < \frac{T-\mu}{\sigma} < 0.2$$
$$-0.6 < X < 0.2$$
$$P(-0.6 < X < 0.2) = \int_{-0.6}^{0.2} N(x)dx = 30.50\%$$

- (Test Scores) Given $N(80,16)$, find probability that $77 < T < 81$

*Same as the previous problem in that you have to transform the problem.*

$$77 < T < 81$$
$$-3 < T - \mu < 1$$
$$-0.75 < \frac{T-\mu}{\sigma} < 0.25$$
$$-0.75 < X < 0.25$$
$$P(-0.75 < X < 0.25) = \int_{-0.75}^{0.25} N(x)dx = 37.21\%$$

*Notice how the previous answer is smaller because the variance is larger. In this problem, more area is concentrated around $T = 80$.*

- (Test Scores) Given $N(90,10)$, find probability that $89 < T < 91$

  *Be careful computing the standard derivation. You will be dividing by $\sqrt{10}$ not 10. Again, we must transform the problem.*

$$\begin{aligned} 89 &< T < 91 \\ -1 &< T-\mu < 1 \\ -0.316 &< \frac{T-\mu}{\sigma} < 0.316 \\ -0.316 &< X < 0.316 \\ P(-0.316 < X < 0.316) &= \int_{-0.316}^{0.316} N(x)dx = 24.80\% \end{aligned}$$

- (Chemical readings in milligrams). Given $N(6,1.69)$, find probability that $5.8 < T < 6.2$.

  *Again, you must transform the problem.*

$$\begin{aligned} 5.8 &< T < 6.2 \\ -0.2 &< T-\mu < 0.2 \\ -0.154 &< \frac{T-\mu}{\sigma} < 0.154 \\ -0.154 &< X < 0.154 \\ P(-0.154 < X < 0.154) &= \int_{-0.154}^{0.154} N(x)dx = 12.24\% \end{aligned}$$

- (Chemical readings in milligrams) Given $N(6,1.69)$, find probability that $T > 6.2$

  *Again, you must transform the problem.*

$$\begin{aligned} T &> 6.2 \\ T-\mu &> 0.2 \\ \frac{T-\mu}{\sigma} &> 0.154 \\ X &> 0.154 \\ P(X > 0.154) = P(X < -0.154) &= \int_{-\infty}^{-0.154} N(x)dx = 43.88\% \end{aligned}$$