Worksheet #13: Derivatives

Spring 2007

Objectives

- Learn about the derivative of exponentials functions.
- Learn about the derivative of the natural logarithm function.

Recall that exponentials have two forms. They are:

$$P = P_0 \cdot a^t \qquad\qquad Q = Q_0 \cdot e^{rt}$$

The derivatives of P and Q are given as:

$$P' = P_0 \cdot \ln(a) \cdot r^t \qquad\qquad Q' = Q_0 \cdot r \cdot e^{rt}$$

Recall that the derivative of a power function is a power function. Notice here that the derivative of an exponential function is exponential.

- What is the derivative of e^t ?

  The derivative of e^t *is* e^t*. This is a very special function.*

- What is the derivative of $200e^t$?

  The derivative of $200e^t$ *is* $200e^t$*.*

Determine the derivative of each of the following functions:

$$N = 1000(1.04)^t \qquad P = 300(0.94)^t \qquad R = 3.45(2.34)^t$$

$$N' = 1000\ln(1.04)(1.04)^t \quad P' = 300\ln(0.94)(0.94)^t \quad R' = 3.45\ln(2.34)(2.34)^t$$

$$A = 100e^{0.025x} \qquad B = 300e^{-1.3t} \qquad C = 300e^{4x}$$

$$A' = 100(0.025)e^{0.025x} = 2.5e^{0.025x} \qquad B' = 300(-1.3)e^{-1.3t} = -390e^{-1.3t} \qquad C' = 300(4)e^{4x} = 1200e^{4x}$$

- What kind of exponential function is N, Growth or Decay ? What is the sign of the $\ln(1.04)$?

  N is exponential growth. Since 1.04 is greater than 1, the function is increasing. The sign of $\ln(1.04)$ *is positive corresponding to the fact that N is increasing.*

- What kind of exponential function is P, Growth or Decay ? What is the sign of the $\ln(0.94)$?

  P is exponential decay. Since 0.94 is less than 1, the function is decreasing. The sign of $\ln(0.94)$ *is negative corresponding to the fact that P is decreasing.*

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The derivative of the natural logarithm function is a power function. We did a problem in Chapter 2 that showed us this result.

$$\frac{d}{dx}\left[\ln(x)\right] = \frac{1}{x}$$

Determine the derivative of each of the following functions:

$$N = 1000\ln(x) \qquad P = 300\ln(t) \qquad R = 3.45\ln(t)$$

$$N' = \tfrac{1000}{x} \qquad P' = \tfrac{300}{t} \qquad R' = \tfrac{3.45}{t}$$

- What is $N'(4)$? What is $N'(10)$? Is N increasing ?

$$N'(4) = 250 \quad N'(10) = 100$$

N is increasing at both of these locations. For any positive x value, $\frac{1000}{x}$ will be positive. Thus, N is always increasing.

- What is $P'(10)$? What is $P'(15)$? Is P concave up or down ?

$$P'(10) = 30 \quad P'(15) = 20$$

P is increasing at both of these locations, but the rate of the increasing is decreasing. Therefore, the second derivative is negative and P would be concave down.

Chain Rule. These problems require using the chain rule. Find the derivative of each of the functions.

$$D = 100e^{4t+2} \qquad E = 300e^{-x^2/17}$$

$$D' = 100e^{4t+2}\cdot 4 \qquad E' = 300e^{-x^2/17}\cdot\left(\tfrac{-2x}{17}\right)$$

$$= 400e^{4t+2} \qquad E' = -\left(\tfrac{600x}{17}\right)e^{-x^2/17}$$

$$A = 1000\ln(3x + 17) \quad C = \ln(t^2 - 4t + 100)$$

$$\begin{aligned} A' &= \tfrac{1000}{3x+17} \cdot 3 \\ &= \tfrac{3000}{3x+17} \end{aligned} \qquad \begin{aligned} C' &= \tfrac{1}{t^2-4t+100} \cdot (2t - 4) \\ C' &= \tfrac{2t-4}{t^2-4t+100} \end{aligned}$$

Homework. From page 152, problems 1-21 odd and application problems 27 and 29.