Worksheet #12: Derivatives

Spring 2007

Objectives

- Learn about the derivative of power functions and polynomials.
- Learn how to prove one case of the power rule.

Determine the derivative of each of the following functions:

$y = x^4 - 30x^2 + 100$ $\qquad z = \sqrt{t} + \sqrt[3]{t}$ $\qquad Q = \frac{200}{t} - \frac{1000}{t^2}$

Each of these requires that each term of the function be put in the stardard form of the power function Ax^p or At^p. Then apply the power function rule. If the problem used radicals, then you final answer should be written with a radial.

$$y = x^4 - 30x^2 + 100$$
$$\frac{dy}{dx} = 4x^3 - 60x$$

$$z = t^{1/2} + t^{1/3}$$
$$\frac{dz}{dt} = \frac{1}{2}t^{-1/2} + \frac{1}{3}t^{-2/3}$$
$$= \frac{1}{2\sqrt{t}} + \frac{1}{3\sqrt[3]{t^2}}$$

$$Q = 200t^{-1} - 1000t^{-2}$$
$$\frac{dQ}{dt} = -200t^{-2} + 2000t^{-3}$$
$$= -\frac{200}{t^2} + \frac{2000}{t^3}$$

$A = (x^2 + 3x - 17)^2$ $\qquad B = (2x + 17)^4$

$$\frac{dA}{dx} = 2(x^2 + 3x - 17)(2x + 3)$$
$$\frac{dB}{dx} = 4(2x + 17)^3(2) = 8(2x + 17)^3$$

Justify each of the answer numerically.

- Is y increasing or decreasing at $x = 2$?

To answer this question, you must use the derivative of y. Plug $x = 2$ into the formula for the derivative.

$$\frac{dy}{dx}(2) = 4 \cdot 2^3 - 60 \cdot 2 = 32 - 120 = -88$$

The deriviatve being negative at $x = 2$ means the function y is decreasing at $x = 2$.

- Is z increasing or decreasing at $t = 1$? and at $t = 2$? What does this tell us about the second derivative of z?

To answer the question of increasing or decreasing, you must use the derivative of z.

$$\frac{dz}{dt}(1) = \frac{1}{2\sqrt{1}} + \frac{1}{3\sqrt[3]{1^2}} = 0.8333$$
$$\frac{dz}{dt}(2) = \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt[3]{2^2}} = 0.5636$$

The function z is increasing at both $x = 1$ and $x = 2$. But notice that the derivatives are decreasing, which implies that the change in the function is slowing down (still increasing, but not as fast). This means the function has a concave down shape, and hence the second derivative is negative.

- Is Q positive or negative at $t = 5$? Is Q increasing or decreasing at $t = 5$

The Q value at $t = 5$ is zero.

$$Q(5) = 200 \cdot 5^{-1} - 1000 \cdot 5^{-2} = 40 - 40 = 0$$

Using the derivative of Q,

$$\frac{dQ}{dt}(5) = -\frac{200}{5^2} + \frac{2000}{5^3} = -8 + 16 = 8$$

So, Q is increasing, since the derivative is positive.

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Prove that if $f(x) = 20x^2$, then $f'(x) = 40x$.

- What is $f(x+h)$? Use FOIL method.

$$f(x+h) = 20(x+h)^2 = 20(x^2 + 2xh + h^2)$$

- What is $f(x+h) - f(x)$?

$$f(x+h) - f(x) = 20(x^2 + 2xh + h^2) - 20x^2 = 20(2xh + h^2)$$

- What is $\frac{f(x+h)-f(x)}{h}$?

$$\frac{f(x+h) - f(x)}{h} = \frac{20(2xh + h^2)}{h} = 20(2x + h)$$

- If h were practically zero, what would $\frac{f(x+h)-f(x)}{h}$ be ?

$$\lim_{h \to 0} 20(2x + h) = 20(2x) = 40x$$

- Notice that you use the slope formula to prove that the instantaneous rate of change of $20x^2$ is $40x$ and is equal to the derivative that the power rule would have given us.

Homework. Practice more.