Worksheet #11: Logistic Function

Spring 2007

Objectives

- Learn a new function that models growth
- Understand the roles of the parameters of the Logistic function
- Understand the behavior of the logistic function - exponential growth with an upper bound

Graph the following function using [xMin,xMax] = [0,52]. Change the [yMin,yMax] so the curve fills your calculator screen.

$$P = \frac{100}{1 + 19e^{-.25t}}$$

Logistic Curve The general form of the logistic function is:

$$P = \frac{L}{1 + C \cdot e^{-kt}}$$

This function models growth. It has characteristics of exponential growth, but has an upper limit to how large the values of P can get. The logistic function can model:

- Population growth in an environment with limited resources
- Sales of a product where the total number of sales is limited
- Patient's response to medication

Properties. Answer the following questions to understand some of the properties of the logistic function and how the parameters control the shape and size of the function.

- Using the general form, identify the parameters L, k, and C for the function that you graphed.

- Determine the initial value of P. What parameters are controlling the initial value of the function?

- Write the exponential function that has the same initial value as P but has continuous rate of 25% growth? Graph this function along with your other graph. Change the $xMax$ to 12 to view functions more closely. Change the $xMax$ back to 52 when done. Which parameter is controlling the initial shape?

- The graph of P is flattening out. What is the upper limit of P? Which parameter is controlling the upper limit?

- An inflection point is a place on the graph where the concavity changes. Use the trace feature to determine the t and P value of the inflection point. How does the P value of the inflection relate to the upper limit? Is your function increasing or decreasing at the inflection? Before the inflection point, is the growth speeding up or slowing down? How about after the inflection point?

- Solve for the t value where P is 80% of the upper limit. Since you are solving for the exponent, you will have to use the natural log. Check your work using the trace feature.

$$80 = \frac{100}{1 + 19e^{-.25t}}$$

Interpretation. If the above equation is modelling record sales, write a summary of the parameter L, the parameter k, the initial value, and the inflection point, and the time to reach 80% of the upper limit in terms of record sales. Assume that P is measured in thousands of records and t is measured in weeks.

Homework. From page 219, Problems 1, 2, 3, 9, and 14.