

Topic Quiz #1 - Functions

Spring 2007

Problem A: For each item, determine the line that passes through the data points.

- A1: (0,1000) and (10,300)

To find the line through these points, you have to first find the slope, then the y-intercept. Also, notice that going from $x = 0$ to $x = 10$, the y-values are decreasing, so you should expect the slope to be negative.

$$m = \frac{1000 - 300}{0 - 10} = \frac{700}{-10} = -70$$

The y-intercept is the y-value when $x = 0$, so we were given that. The y-intercept is 1000. Final answer:

$$y = -70x + 1000$$

- A2: (30,100) and (60,2500)

To find the line through these points, you have to first find the slope, then the y-intercept. Also, notice that going from $x = 30$ to $x = 60$, the y-values are increasing, so you should expect the slope to be positive.

$$m = \frac{2500 - 100}{0 - 10} = \frac{800}{10} = 80$$

Notice that moving from $x = 60$ to $x = 30$, the y-values dropped by 2400. So, moving from $x = 30$ to $x = 0$, the y-values should also drop by 2400, so at $x = 0$, $y = -2300$. Final answer:

$$y = 80x - 2300$$

We could have also used the point-slope formula to determine the line.

$$y - 100 = 80(x - 30)$$

$$y - 100 = 80x - 2400$$

$$y = 80x - 2300$$

Problem B: You are going to throw a March Madness Basketball party and plan on serving soft drinks and pizza rolls. Soft drinks can be bought for \$1.25 per bottle and pizza rolls cost \$4.00 per bag. You have \$40 to spend on the party for these items.

- Write a formula describing the cost if you bought S bottles of soda and P bags of pizza rolls.

$$Cost = 1.25S + 4P$$

- Write the equation that incorporates your budget constraint.

Here, we let the Cost equal (or perhaps less than or equal) to \$40. So,

$$40 = 1.25S + 4P$$

We can solve for one of the variables and get,

$$P = 10 - \frac{5}{16}S$$

But, to graph a line, we only need two points. So, we can get the intercepts by setting one of variable to zero, then solving for the other variable. This is done easily using the first equation.

$$40 = 1.25S + 4 \cdot 0$$

$S = 32$ bottles of soda when there are no pizza rolls

$$40 = 1.25 \cdot 0 + 4P$$

$P = 10$ bags of pizza rolls where there are no sodas

- Graph the equation from above. Compute the horizontal and vertical intercepts and label them on your graph. Interpret what these intercepts mean with respect to your party.

Draw two axes (S and P). Label the points ($S = 0, P = 10$) and ($S = 32, P = 0$). Draw the line between the two points. The intercepts are the maximum number of items that you can buy when you don't buy any of the other item.

Topic Quiz #1 - Functions

Spring 2007

Problem C: You win the lottery. What is the present value of your winnings if the state is going to pay you in three payments: \$10,000 (today), \$20,000 (in five years), and \$40,000 (in ten years). Assume you can earn 6% compounded annually on your money.

You are given three payments. One of these is given to you today - in the present. Its present value is $P_1 = \$10,000$. But the other two payments are given to you in the future. You must figure each of their present values using the formula:

$$F = P(1.06)^t$$

$$20000 = P_2(1.06)^5 \quad 40000 = P_3(1.06)^{10}$$

Notice that the question stated that the interest was compounded annually, not continuously. So, the present values of the payments are $P_1 = \$10,000$, $P_2 = \$14,945.16$, and $P_3 = \$22,335.79$. The present value of all three payments is the sum of present values of each payment. Final answer is \$47,280.95.

It might look like you are receiving \$70,000, but that is not what it is worth.

Problem D: Solve for t . You must use the natural log in the process - guessing is not

permitted. Double check your work by plugging back into the original equation.

$$3000(1.04)^t = 1000(1.10)^t$$

For the right t -value, both sides will be the same number. If you take the natural log of both sides, both sides will still equal each other.

$$\ln(3000(1.04)^t) = \ln(1000(1.10)^t)$$

$$\ln(3000) + t \ln(1.04) = \ln(1000) + t \ln(1.10)$$

What two rules did we use here? Now, solve for t .

$$\ln(3000) - \ln(1000) = t(\ln(1.10) - \ln(1.04))$$

$$t = 19.59$$