the average was 45 and the standard deviation was 3 (assume the test scores are from a normal distribution)?

- *Bell shaped curve*

$$N(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$$

- *Shading in the region that is being requested*
- *Convert random variable from $N(45, 9)$ to $N(0, 1)$*
- *State the question as a probability inequality*
- *Expressing the answer as an integral expression and compute the integral.*

$T \sim N(45, 9)$

$P(T > 40) =$

$P(T - 45 > -5) =$

$P\left(\frac{T-45}{3} > -\frac{5}{3}\right) =$

$\int_{-5/3}^{\infty} N(x)\,dx =$

$\longrightarrow \int_{-5/3}^{0} N(x)\,dx + \int_{0}^{\infty} N(x)\,dx =$

$.4522 + .5 =$

$.9522 = 95.22\%$

- The values of $y = 8\sqrt{t}$ range from 0 to 24 for t values that range from $t = 0$ to $t = 9$. What is the average y-value over this interval?

  - *What is the picture that goes along with this problem?*
  - *Formula for average value.*
  - *How was this formula derived ? How is it similar to taking the average of 8 test scores?*
  - *Express the answer as an integral expression and compute the integral.*

Avg Value of f over $[a, b]$ $= \frac{1}{b-a} \int_a^b f(t)\,dt$

Avg Value of $8\sqrt{t}$ over $[0, 9]$ $= \frac{1}{9} \int_0^9 8\sqrt{t}\,dt$

$= \frac{1}{9}(144.) = 16$

- If a train pulls out of a station and its velocity is measured to be $V = 100(1 - e^{-0.05t})$ feet per second. What is the speed of the train after two minutes ? How far has the train travelled in the first two minutes? What was its average speed during this time?

  - *Given velocity and you want to know distance. In high school, you might have used the formula of Rate times Time equals distance. How does this relate to this problem?*
  - *What is the picture that goes along with this problem?*
  - *Express the answer as an integral expression.*
  - *What does the expression $\int_0^T V(t)dt$ mean ?*
  - *What is you were given a formula for the distance that an object has travelled? $H = -16t^2 + 300t + 5$ is the height of a ball that is thrown into the air. What does $H'(t)$ tell you? Remember every function tells a story.*

$$\text{Distance} = \int_0^T V(t)\,dt$$

$$\text{Distance} = \int_0^{120} 100\left(1 - e^{-.05t}\right) dt$$

$$= 10004.958 \text{ ft}$$

- If you have \$3000 and it can grow at a continuous rate of 6%, then how much do you have after two years? How fast is it growing after two years? How fast has your money growing in the first two years?

  - *Again, every function tells a story. What is the function that tells you how much money you have? What does the derivative of this function tell you?*
  - *What is the difference between the average rate of change and the instantaneous rate of change?*

$$P.V = 3000\, e^{(.06 \cdot 2)} = 3382.49 \text{ \$}$$

$$P = 3000\, e^{.06t}$$

Instantaneous $\longrightarrow$ $P' = 180\, e^{.06t}$ $\quad P'(2) = 180\, e^{.12} = 202.95 \text{ \$/yr}$

Average over ~~02.~~ two years $= \dfrac{P(2) - P(0)}{2} = 191.25 \text{ \$/yr}$

- If the risk of failing an exam is a function of hours of study time, determine the optimal amount of time to study for a test if risk $R = 4t + \frac{49}{t}$ where t is in hours.

  - *Graph this function.*
  - *What does the optimal amount of time mean in this problem?*
  - *Is the optimal time a local min or local max? What is the mathematical statement that can be made about the optimal time (call it t*)?*
  - *What are the steps to finding the optimal time? Carry them out.*
  - *What is the risk of failing if you study the optimal amount of time?*
  - *If you find the second derivative of R and plug in the optimal time, what does this answer tell you about the optimal time?*

$R = 4t + 49t^{-1}$

$\frac{dR}{dt} = 4 - \frac{49}{t^2}$

$\frac{dR}{dt} = 0$ when $4 = \frac{49}{t^2}$

$t = \frac{7}{2} = 3.5$ ~~h~~ hours.

- If the amount of medicine in your bloodstream is given by the function $M = 1000te^{-0.01t}$ where t is the number of minutes since you took the medicine, then determine when you have the most amount of medicine in your bloodstream? How much medicine is in your bloodstream at that time?

  - *Graph this function.*
  - *What does the optimal amount of time mean in this problem?*
  - *Is the optimal time a local min or local max? What is the mathematical statement that can be made about the optimal time (call it t*)?*
  - *What are the steps to finding the optimal time? Carry them out.*

- *If you find the second derivative of R and plug in the optimal time, what does this answer tell you about the optimal time?*

$M = 1000\, t\, e^{-.01t}$

Use product rule

$\frac{dM}{dt} = 1000\, e^{-.01t} - 10\, t\, e^{-.01t}$

$= 10 e^{-.01t}\left[\, 100 - t \,\right]$

$\frac{dM}{dt} = 0$ when $t = 100$ minutes.

- Find the vertex of the parabola $y = 10x^2 - 6000x + 350$

  - *Draw any parabola. What are some of the properties that are assoicated with the vertex of the parabola?*
  - *What are the steps to find the vertex?*
  - *What does the second derivative tell you about the parabola?*

$y = 10x^2 - 6000x + 350$

$\frac{dy}{dx} = 20x - 6000$

$\frac{dy}{dx} = 0$ when $20x = 6000$

$x = 300$

$\frac{d^2y}{dx^2} = 20$

$\frac{d^2y}{dx^2} > 0$ means the parabola is concave up.

$x = 300$ is a minimum.

- Find the local maximum of that exist between $x = 0$ and $x = 10$ of the function $y = x(x-10)^2$.

  - *What rules do you use to take the derivative of this function?*
  - *Graph this function. You can see that there is a maximum that lies between $x = 0$ and $x = 10$. How can you find this point exactly?*
  - *Name the root(s) of this function. Define roots.*

- *Name the critical point(s) of this function. Define critical points.*
- *Name the inflection point(s) of this function. Define inflection points.*

$$y = x(x-10)^2$$

Product Rule

$$\frac{dy}{dx} = (x-10)^2 + 2x(x-10)$$
$$= (x-10)[x-10+2x]$$
$$= (x-10)[3x-10]$$
$$= 3x^2 - 40x + 100$$

$$\frac{d^2y}{dx^2} = 6x - 40$$

Roots : $x=0$ $x=10$

Critical Pts : $x=10$ $x=\frac{10}{3}$

Inflection Pts : $x = \frac{20}{3}$

- During the summer you plan to work at a candy factory as a quality control officer. Your manager says your job will be to pick the candies off the production line that look flawed. He says there are usually about 10 bad pieces of candy for every 2000 candies produced. You were wondering what are the chances of seeing at least 250 good pieces of candy in a row off the production line. What is the probablity that this might happen?

  - *What type of probability problem is this?*
  - *What type of graph is assoicated with this problem?*
  - *How do you determined the function to graph for this problem?*
  - *What are some of the properties that you know about the graph of a probability density function?*
  - *Express the answer as an integral. Compute this integral.*

$\lambda = 200$ ~~candies~~ candies (average waiting time)

$$pdf = \frac{1}{200} e^{-w/200}$$

$$P(w \geq 250) = 1 - P(w \leq 250) = 1 - \int_0^{250} \frac{1}{200} e^{-w/200} \, dw$$

$$= 1 - .7135$$
$$= .2865$$
$$= 28.65\%$$