Stephanie B Alexander (sba@math.uiuc.edu), Department of Mathematics, 273 Altgeld Hall, 1409 West Green Street, Urbana, IL 61801, and Richard L Bishop* (bishop@math.uiuc.edu), Department of Mathematics, 273 Altgeld Hall, 1409 West Green Street, Urbana, IL 61801. Spines and Topology of Thin Riemannian 3-Manifolds. Preliminary report.
Lagunov and Fet constructed a 3-dimensional Euclidean domain $M$ whose boundary is a 2 -sphere with normal curvature $\left|\kappa_{\partial M}\right| \leq 1$ and whose inradius is $\sqrt{3 / 2}-1+\epsilon \approx .225$ for every $\epsilon>0$. We consider abstract Riemannian manifolds $M$ of sectional curvature $\left|K_{M}\right| \leq 1$, with simply connected boundary such that $\left|\kappa_{\partial M}\right| \leq 1$, and with inradius $<.108$. Then the cut locus of the boundary has been proved by us in a previous paper (Advances in Mathematics, 155, 23-48, (2000)) to be a 3 -branched simple polyhedral spine of $M$. In dimension 3, these spines are fake surfaces. In this case we classify the possibilities for $M$ up to homeomorphism, namely, they are connected sums of $p$ copies of $P^{3}, t$ copies of the lens spaces $L(3, \pm 1)$, and $\ell$ handles either $S^{2} \times S^{1}$ or the twisted $S^{2}$ bundle over $S^{1}$, with $\beta$ 3-balls removed, where $p+t+\ell+\beta \geq 2$. By modifying the example cited above we show that our inradius bound is sharp up to a factor of 2. (Received January 23, 2001)

