EXTREMAL APPROXIMATELY CONVEX FUNCTIONS AND ESTIMATING THE SIZE OF CONVEX HULLS

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ABSTRACT. A real valued function f defined on a convex K is an ***approximately convex function*** iff it satisfies

$$f\Big(\frac{x+y}{2}\Big) \le \frac{f(x)+f(y)}{2} + 1.$$

A though study of approximately convex functions is made. The principal results are a sharp universal upper bound for lower semi-continuous approximately convex functions that vanish on the vertices of a simplex and an explicit description of the unique largest bounded approximately convex function E vanishing on the vertices of a simplex.

A set A in a normed space is an ***approximately convex set*** iff for all $a, b \in A$ the distance of the midpoint $(a+b)/2$ to A is ≤ 1. The bounds on approximately convex functions are used to show that in $\mathbf{R}^n$ with the Euclidean norm, for any approximately convex set A, any point z of the convex hull of A is at a distance of at most $[\log_2(n-1)] + 1 + (n-1)/2^{[\log_2(n-1)]}$ is from A. Examples are given to show this is the sharp bound. Bounds for general norms on $\mathbf{R}^n$ are also given.

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