

Mathematics 700 Homework  
Due Wednesday, September 3

- (1) Let  $V$  be a vector space and  $v_1, \dots, v_m \in V$  with the property that if any vector is removed from  $\{v_1, \dots, v_m\}$  then the resulting set has a smaller span than  $\{v_1, \dots, v_m\}$ . Then  $v_1, \dots, v_m$  is linearly independent.

**Restatement:** If  $S \subset V$  is a finite subset of  $V$  such that for every proper subset  $S' \subset S$  we have  $\text{Span}(S') \neq \text{Span}(S)$ , then  $S$  is linearly independent.

HINT: One way would be to use the proposition on page 28 of the notes and a proof by contradiction.

- (2) A vector space  $V$  is **finite dimensional** iff there it is spanned by a finite subset. (That is  $V$  is finite dimensional iff there are  $v_1, \dots, v_m \in V$  such that  $V = \text{Span}\{v_1, \dots, v_m\}$ . Show that any finite dimensional vector space is spanned by a linearly independent set.