Mathematics 700 Quiz #1

Name:______________________________

1. Give an example of two linear equations with real coefficients in two unknowns so that
   (a) They have exactly one solution.

   (b) The set of solutions is a line in $\mathbf{R}^2$.

   (c) They have no solutions.

2. Find all solutions to

$$\begin{aligned} 2x + y + z &= 5 \\ x + y - 4z &= 3 \end{aligned}$$

3. The matrix

$$A = \begin{bmatrix} 6 & 1 & -4 & 6 & 9 \\ 2 & 1 & -3 & -5 & 1 \\ 2 & -1 & 4 & 4 & 5 \end{bmatrix}$$

has row canonical form

$$\begin{bmatrix} 1 & 0 & 0 & 5/4 & 7/4 \\ 0 & 1 & 0 & -51/2 & -11/2 \\ 0 & 0 & 1 & -6 & -1 \end{bmatrix}.$$

Use this to find all solutions to

$$\begin{aligned} 6x + y - 4z + 6w &= 9 \\ 2x + y - 3z - 5w &= 1 \\ 2x - y + 4z + 4w &= 5. \end{aligned}$$

4. Let B be an m by n matrix with $m < n$ (that is B has more more collumns than rows). Then explain why there is a nonzero column vector $x \in \mathbb{F}^n$ so that $Bx = 0$.