Mathematics 552 Test #2 Name: ______________________________

Show your work! Answers that do not have a justification will receive no credit. Also put your answers in some standard "human" form such as $x + iy$ or polar form, and not some specialized form used by your calculator.

(1) (30 Points) Compute the following:

(a) $\mathrm{Arg}(11 - 11i)$ ______________________

(b) $\mathrm{Log}(11 - 11i)$ ______________________

(c) The principle branch of i^i ______________________

(d) $\displaystyle\int_C z^7\,dx$ where C is the curve $z(t) = 1 + ti$ with $0 \le t \le 1$ and simplify your answer. ______________________

(e) $\displaystyle\int_{|z|=3} \frac{e^z \cos(z)}{z}\,dz$ ______________________

(f) $\displaystyle\int_{|z|=7} \frac{z^3 e^z}{z+1}\,dz$ ______________________

(g) $\displaystyle\int_{|z-i|=1/2} \frac{z^3}{z^2+1}\,dz$ ____________________

(h) $\displaystyle\int_{|z|=2} (-y\,dx + x\,dy)$ ____________________

(2) (10 points) Define the following

(a) D is a ***domain***.

(b) D is a ***simply connected domain***.

(c) C is a ***closed curve***.

(d) C is a ***simple closed curve*** .

(e) $\mathrm{Arg}(z)$ where $z \neq 0$.

(3) (15 points)

(a) State Green's theorem.

(b) State the Cauchy-Riemann equations.

(c) Use Green's Theorem and the Cauchy-Riemann equations to show that if D is a bounded domain with nice boundary, and $f(z)$ is a function analytic in D and continuous on $D \cup \partial D$ that

$$\int_{\partial D} f(z)\, dz = 0.$$

(4) (15 points)

(a) State the Cauchy Integral Formula.

(b) Prove the Cauchy Integral Formula.

(5) (10 Points)

(a) Define antiderivative.

(b) Let D be the disk $\{z : |z - 3| < 4\}$. Explain why the function $f(z) = e^{z^2} \cos(z^3)$ has an antiderivative in D. (This should not involved much more than quoting a theorem and saying why its hypothesis hold.)

(6) (10 points) Let D be the disk $\{z : |z| < 1\}$. Explain why there is an analytic function $h(z)$ with $h(z)^2 = 6 + z^3$.

(7) (10 points) Explain why the function analytic function $f(z) = \dfrac{1}{z-1}$ does not have an antiderivative in the domain $D = \{z : 1/3 < |z-1| < 4\}$. (This should involve using some English.)