Mathematics 552 Test #1 Name:____________________________

Show your work! Answers that do not have a justification will receive no credit.

(1) (30 Points) Compute the following:

(a) $(1+2i)(3+4i)$ ____________________

(b) $\dfrac{1+2\,i}{3+4\,i}$ ____________________

(c) $\arg(-1+\sqrt{3}\,i)$ ____________________

(d) $(-1+\sqrt{3}\,i)^{11}$ ____________________

(e) $\left|\dfrac{(a+bi)^9}{(a-bi)^8}\right|$ ____________________

(f) The first four terms (that is up to and including the z^3 term) of the series for $\dfrac{e^z-1-z}{z^2}$.

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(2) (5 points) Find a Möbius transformation $f(z)$ such that $f(i) = 0$, $f(2) = 1$, and $f(-i) = \infty$.

$f(z) =$ ____________________

(3) (5 points) Use the definition $\sinh(z) = \dfrac{e^z - e^{-z}}{2}$ to show that $\sinh(iz) = i\sin(z)$.

(4) (10 Points) Find all values of $(-8)^{\frac{2}{3}}$. ____________________

(5) (10 Points) Solve $\cos(z) = \dfrac{5}{4}$. ____________________

(6) (10 Points) Find the image of $|z-i|=1$ under the map $f(z)=\dfrac{1}{z}$. Draw a picture of both $|z-i|=1$ and its image.

(7) (10 Points) Find power series for $f(z)=(1+z)^{-3}$.

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(8) (10 Points)

(a) Let $U \subseteq \mathbb{C}$ be an open set and $f\colon U \to \mathbb{C}$ a function. State the definition, in terms of a limit, of what if means for f to be ***analytic***.

(b) If $f(z) = u(x, y) + iv(x, y)$ state the ***Cauchy-Riemann equations***.

(c) Give the derivation of the Cauchy-Riemann equations.

(9) (5 points) Show that an analytic function $f(z)$ with $\operatorname{Re} f(z) = 5$ is constant.

(10) (10 points) Let U be the domain defined by the inequalities $1 < |z| < 2$, and $0 < \arg(z) < \frac{\pi}{2}$.

(a) Draw a picture of U.

(b) Find the image of U under the map $f(z) = z^3$ and draw its picture.

Extra Credit: (5 points) Show that $f(z) = z^2 + 4z$ is one to one on the disk $|z| < 2$.