Mathematics 552, Review for Test 2

Reminder: The test is Monday, March 20.

The best thing to do is to go over the quizzes and homework. I will look at these while making up the test. You can find these at

`http://www.math.sc.edu/~howard/Classes/552d/`

(1) Left over from the first test. There are some topics that you will certainly still have to know from the first part of the course. This include.

   (a) The definition of an ***analytic function*** and that the relation of being analytic to the ***Cauchy-Riemann*** equations.

   (b) The basic transcendental functions. That is e^z, $\cos(z)$, and $\sin(z)$ (see the review sheet for test one).

   (c) Related to this is that we know have $\arg(z)$, $\operatorname{Arg}(z)$, $\log(z)$, $\operatorname{Log}(z)$, and z^α. You should be able to compute with these know ***principle branch*** means when applied to these functions. Know where these functions are analytic and what their derivatives are.

(2) Line integral and related concepts.

   (a) Be able to compute line integrals of the form $\int_C P\,dx + Q\,dy$ where C is a curve. This may include having to parameterize this curve C. Be able to parameterize a line segment between two points and a circle of radius r centered at a point z_0.

   (b) Know the statement of ***Green's Theorem.***

(3) Cauchy's Theorem and related topics.

   (a) Be able to use Green's theorem and the Cauchy-Riemann equations to show that if D is a bounded domain with nice boundary, $f(z)$ is analytic in D and continuous on $D \cup \partial D$ then $\int_{\partial D} f(z)\,dz = 0$.

   (b) Know the definitions of ***simply connected domain*** and ***closed curve***, and ***simple closed curve***.

   (c) Know statement of Cauchy's theorem in the form
   Cauchy's Theorem: *If D is a simply connected domain, and $f(z)$ is analytic in D, then*

$$\int_C f(z)\,dz = 0.$$

   (d) ***Independence of Path Theorem:*** *If D is simply connected and $f(z)$ is analytic in D the if C_1 and C_2 are curves in D with the same initial point and same end points, then*

$$\int_{C_1} f(z)\,dz = \int_{C_2} f(z)\,dz.$$

   (e) ***Existence of Antiderivatives:*** *If $f(z)$ is analytic in the simply connected domain D, then there is an analytic function $F(z)$ in D with $F'(z) = f(z)$.*

   (f) Be able to give an example of a domain D and a functions $f(z)$ that does not have an antiderivative in D and be able to explain why it does not. (For example $D = \{z \in \mathbf{C} : z \neq 0\}$ and $f(z) = 1/z$ works.)

(g) **Existence of Logarithms:** *If the $f(z)$ is analytic and non-vanishing in the simply connected domain D then there is an analytic function $g(z)$ in D with $e^{g(z)} = f(z)$.*

(h) **Existence of Roots:** *Let D be a simply connected domain, $f(z)$ an analytic function $f(z)$ that does not vanish in D, and $n \neq 0$ an integer. Then there is an analytic function $h(z)$ in D with $h(z)^n = f(z)$.* You should be able to prove this using the existence of logarithms.

(4) The Cauchy integral formula and related topics.

(a) Know the statement of
Cauchy Integral formula: *If D is a bounded domain with nice boundary and $f(z)$ is a function that is analytic in D and continuous on $D \cup \partial D$, then for $z \in D$*

$$f(z) = \frac{1}{2\pi 1} \int_{\partial D} \frac{f(\zeta)\, d\zeta}{\zeta - z}.$$

(b) Know the proof of the Cauchy Integral Formula.

(c) Know how to use the Cauchy Integral Formula to evaluate integrals as we did on the last homework assignment.

(d) Know the statement of the basic estimate for integrals.
Basic Integral estimate: *If $f(z)$ is defined along a curve C and $|f(z)| \leq M$ on C, then*

$$\left| \int_C f(z)\, dz \right| \leq M \operatorname{Length}(C).$$

(5) As usual there will be ***surprise mystery questions***.