Mathematics 552 Homework due Friday, February 3 , 2006

We now know that the complex exponential e^z is given by

$$e^z = e^{x+iy} = e^x(\cos(y) + i\sin(y))$$

Now this implies that if θ is real, that

$$e^{i\theta} = \cos(\theta) + i\sin(\theta) = \operatorname{cis}(\theta)$$

which justifies the book's notation of $e^{i\theta}$ for $\operatorname{cis}(\theta)$.

Use this to show that

(1) $|e^z| = e^{\operatorname{Re} z}$.

(2) More generally the polar form of e^z is

$$e^z = e^{\operatorname{Re} z} e^{i \operatorname{Im} z} = e^x e^{iy}.$$

(3) Find all solutions to $e^z = 1$.

(4) Find all solutions to $e^z = 2$.

(5) Find all solutions to $e^z = -3i$.

(6) Find all solutions to $e^z = 1 + i$.

We have (or at least will see shortly) that

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2}, \qquad \sin z = \frac{e^{iz} - e^{-iz}}{2i}. \tag{1}$$

(7) Use (1) to find all solutions to $\cos(z) = 2$. Hint: Set $\cos(z) = \frac{e^{iz}+e^{-iz}}{2} = 2$. Multiply through by e^{iz}. The result is a quadratic equation in e^{iz}. Solve for e^{iz} and then solve the results for z.

Quiz on Friday: Know the formulas

$$e^{-z} = \cos(z) + i\sin(z), \qquad \cos(z) = \frac{e^{iz} + e^{-iz}}{2}, \qquad \sin z = \frac{e^{iz} - e^{-iz}}{2i}.$$