Mathematics 552 Homework due Wednesday, February 1 , 2006

(1) Assuming that you can different term by term find the sum of

$$S = 1 + 2r + 3r^2 + 4r^3 + \cdots = \sum_{n=1}^{\infty} n r^{n-1}$$

by taking the derivative of the series

$$1 + r + r^2 + r^3 + r^4 + \cdots = \sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$$

(2) We know that

$$S = 1 + r + r^2 + \cdots + r^n = \sum_{k=0}^{n} r^k = \frac{1}{1-r} - \frac{r^{n+1}}{1-r}$$

(a) Take the derivative of this to get a formula for the sum

$$S' = 1 + 2r + 3r^2 + \cdots + nr^{n-1} = \sum_{k=0}^{n} k r^{k-1}.$$

(b) Take another derivative to get a formula for the sum

$$S'' = 2r + 2 \cdot 3r^2 + +3 \cdot 4r^2 \cdots + (n-1)nr^{n-2} = \sum_{k=0}^{n} (k-1)kr^{k-2}.$$

(c) Use this formulas, or any other method you like, to show that if $|r| < 1$ the sums

$$\sum_{k=0}^{\infty} k r^{k-1} \qquad \text{and} \qquad \sum_{k=0}^{\infty} (k-1)kr^{k-2}.$$

both converge.

Quiz on Wednesday: Have the follow series memorized. The ***binomial expansion***:

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}, \qquad \text{where} \qquad \binom{n}{k} = \frac{n!}{k!(n-k)!}.$$

The series for e^x

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}.$$

The series for a finite geometric series

$$a + ar + ar^2 + \cdots + ar^n = \sum_{k=0}^{n} ar^k = \frac{a - ar^{n+1}}{1-r}.$$

The series for $\sin(x)$ and $\cos(x)$.

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}.$$

$$\cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}.$$