Mathematics 552 Test #2 Name:______________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (12 Points) Define the following:
   (a) f is analytic in the open set U.

   (b) The function f is entire.

   (c) The function ϕ is harmonic in the open set U.

   (d) The functions u and v are conjugate harmonic functions in the open set U.

2. (8 Points) Find the derivatives of the following:
   (a) $f(z) = \exp(1 + z^2)$. ________________________

   (b) $h(z) = z(\sin(z) + \cos(3z))^4$. ________________________

3. (15 Points) Compute the following.

(a) $\mathrm{Log}(\pi\, i)$ ____________________

(b) All values of $(3i)^i$ ____________________

(c) $\sin(1+2\,i)$ (put the answer in the form $a+b\,i$). ____________________

4. (10 Points) Let α be a complex number and let $f(z) = z^\alpha$ be the principle branch of z^α.

(a) Give the definition of $f(z)$ in terms of the exponential and Log.

(b) Use this definition to find a formula for $f'(z)$. (Simplify your answer.)

5. (10 Points) Find all the harmonic conjugates to $u = x^3 - 3xy^2 - y$.

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6. (15 Points) Let $f(z) = (x^2 - y^2 - 4xy - y) + (2xy + 2x^2 - 2y^2 + x)\, i$

(a) Show is $f(z)$ is entire.

(b) Compute the derivative of $f(z)$.

7. (10 Points) Find all solutions to $\tan(z) = -2\, i$

8. (10 Points) Let $f = u + v\,i$ be a complex valued function which is analytic on the open subset U of $\mathbb{C}$.
   (a) State the Cauchy-Riemann equations for f.

   (b) (10 Points) Use the Cauchy-Riemann equations for f to show that $u = \mathrm{Re}(f)$ is harmonic.

9. (10 Points) If $f(z)$ is an entire function so that $\mathrm{Im}(f)$ is constant, then show that $f(z)$ is constant.