

**Mathematics 522 In Class Portion of Final** Name: \_\_\_\_\_

**Show your work!** Answers that do not have a justification will receive no credit.

1. (15 Points) State or define the following:

(a) The principle value of  $\text{Arg}(z)$ .

(b) The Cauchy integral theorem.

(c) The Cauchy integral formula.

(d) The Cauchy-Riemannian equations.

2. (5 Points) Let  $f = u + i v$  be analytic in the domain  $D$ . Then use the Cauchy-Riemannian equations to show that  $u$  is harmonic in  $D$ .

3. (20 Points) Compute the following

(a)  $e^{\frac{-\pi}{3}i}$

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(b)  $(1 + i)^{10}$

(c)  $\arg(\sqrt{3} - i)$

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(d)  $\log(4 - 4i)$

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(e) The principle value of  $i^{2i}$

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(f) all the cube roots of  $-27$ .

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4. (10 Points) Find the harmonic conjugates of  $u = 2x^2 - 2y^2 - 6xy + 2x + y$
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*Have a nice summer!*