Mathematics 552
Take Home Portion of Final

This is due in at the time of the in class portion of the final, Wednesday, May 5, 9:00 AM. The same rules apply as on the last take home test. Here is a somewhat edited version of those rules. You may work together on this test, but of course this does not mean just copying an answer that someone has done. Here are two methods of collaboration that I have found useful.

1. Start on a problem together and work on it jointly in a small group (two or three, five is getting to be too large). I find this easiest at a black board (no spectators, everyone standing at the board). Others find it easier working a table working on a pad in such a way that everyone can see what is going on.
2. Work on a problem separately and then compare answers and methods. Make sure that each person involved understands what the others are going and why it is right or wrong.

As you have several days to do the exam I expect the results of be well written and complete. ***Complete means more English, not necessarily more algebra.*** It will not bother me if you say "it now follows by a calculation that" and skip a little of the algebra. (For example something like "Then we have $\lambda^2 - 2\lambda + 2 = 0$ and so by the quadratic formula it follows that $\lambda = 1 \pm i$" is fine.) But you should explain what you are computing and what it has to do with the solution to the problem. One good way to do this is to exchange first drafts of a problem with someone else that has done the problem and have them make comments on where they had trouble reading your solution while you make a similar comments on their paper. You don't have to end up agreeing what is the "best" way to write things, but if someone is having trouble reading what you have written then it probably means that you needed to do some rewriting.

1. (10 Points) Find all solutions to $\cos(z) = i$.
2. (10 Points) Let n be a positive integer and a, b any two complex numbers. Show that $\max_{|z|\le 1} |az^n + b| = |a| + |b|$.
3. (10 Points) Let u be a harmonic function defined on the domain D. Show that the function $f = u_x - iu_y$ is analytic.
4. (10 Points) Find the real and imaginary parts of $\tan(x + iy)$.
5. (20 Points) Let $f(z) = \dfrac{az+b}{cz+d}$ where a, b, c, d are complex numbers with $ad - bc \neq 0$. Let $C_r(z_0)$ be the circle of radius r about z_0. Show that the image of $C_r(z_0)$ by f is either a circle or a

straight line. (Recall that the image of $C_r(z_0)$ by f is the set $f[C_r(z_0)] = \{f(z) : z \in C_r(z_0)\}$.)

6. (20 Points) Let $P(z)$ be a polynomial which has no zeros on the simple closed positively oriented curve Γ. Prove that the number of zeros of $P(z)$ (counting multiplicity) *inside* of Γ is given by the integral
$$\frac{1}{2\pi i}\oint_\Gamma \frac{P'(z)}{P(z)}dz.$$
(Hint: This is problem 24 on page 169 of the text which has a very helpful hint. Also if $P(z)$ has n roots then we have shown in class that $P(z) = a_n(z-z_1)(z-z_2)\cdots(z-z_n)$ where $z_1, \ldots, z_n$ are the zeross of $P(z)$ and $a_n \neq 0$. You may use this form of $P(z)$.)
7. (20 Points) Use the Residue Theorem to compute the following integrals.

(a) $\displaystyle\oint_{|z|=5} \frac{z}{z^2-9}\,dz,$ (b) $\displaystyle\oint_{|z|=4} \frac{e^z}{z^2+2z-15}\,dz.$

What to expect on the in class part of the final.

The in class part of the final will be, apart from various and sundry surprise mystery questions, mostly calculations similar to ones that have appeared on earlier exams and quizzes. You should also know the statements of the major theorems we have covered and be able to define the important terms. In terms of topics covered since the third exam, know the definition of a residue and the statement of the Residue Theorem.