Mathematics 550 Test #2 Name:______________________________

1. (30 points) Complete the following identities:

   (a) $\nabla(fg) =$

   (b) $\operatorname{div}(f\mathbf{F}) =$

   (c) $\operatorname{div}(\mathbf{F} \times \mathbf{G}) =$

   (d) $\operatorname{div}\operatorname{curl}\mathbf{F} =$

   (e) $\operatorname{curl}(f\mathbf{F}) =$

   (f) $\dfrac{d}{dt}\big(\mathbf{b}(t) \times \mathbf{c}(t)\big) =$

2. (10 points) What are the velocity and acceleration of the path $\mathbf{c}(t) = (t, t^2, t^3)$?

   Velocity = __________________________

   Acceleration= __________________________

3. (10 points) Sketch the graph of the curve parameterized by $x(t) = 3\cos(t)$ and $y(t) = 2\sin(t)$.

4. (15 points) Let $f(x, y) = x^2 - xy + y^3$.
(a) What the equation of the tangent to $z = f(x, y)$ at the point $(1, 2, 7)$?

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(b) Where does the tangent plane intersect the z-axis?

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5. (10 points) What is the tangent line to $\mathbf{c}(t) = (t^2, t^3)$ when $t = 2$? ________________________

6. (5 points) Let $f = xy + yz + xz$. Then compute the gradient of f.

$\nabla f =$ ________________________

7. (10 points) Let $\mathbf{F} = yz\mathbf{i} + xz\mathbf{j} + \mathbf{k}xy^2$. Then compute $\operatorname{curl}\mathbf{F}$.

$\operatorname{curl}\mathbf{F} =$ ________________________

8. (10 points)

(a) Let $V\colon \mathbf{R}^3 \to \mathbf{R}$ be a function and $\mathbf{c}\colon [a, b] \to \mathbf{R}^3$ a path. Then state the chain rule for

$$\frac{d}{dt}V(\mathbf{c}(t)) =$$

(b) Now assume that $\mathbf{c}(t)$ satisfies

$$m\mathbf{c}''(t) = -\nabla V(\mathbf{c}(t))$$

for a positive number m (the "mass"). The show that

$$E = \frac{1}{2}m\|\mathbf{c}(t)\|^2 + V(\mathbf{c}'(t))$$

is constant.