Mathematics 550 Test #1   Name: _____
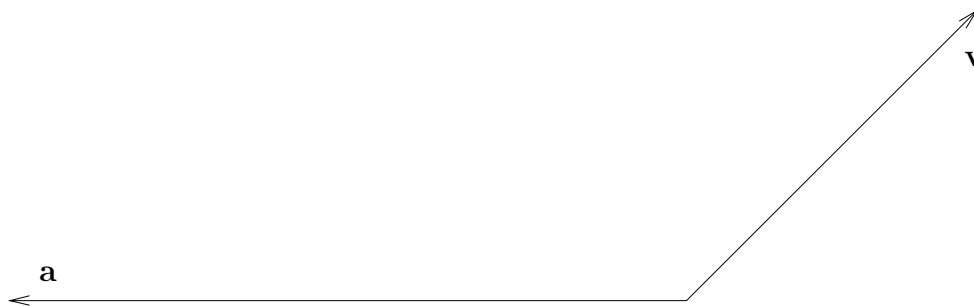
1. (5 points) In the figure draw and label the vector $\mathrm{proj_a(v)}$

2. (5 points) What is the area of the triangle with vertices $P = (1, 2, 3)$, $Q = (-2, 4, 7)$ and $R = (4, 2, -1)$?

3. (25 points) Let $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k}$, $\mathbf{b} = \mathbf{i} + 4\mathbf{j} - 2\mathbf{k}$, $\mathbf{v} = 4\mathbf{i} + 2\mathbf{j} + \mathbf{k}$. Then compute

(a) $\mathbf{a} + \mathbf{b}$

(b) $2\mathbf{a} - 4\mathbf{b}$

(c) $\mathbf{a} \cdot \mathbf{b}$

(d) $\|\mathbf{a}\|$

(e) the angle between $\mathbf{a}$ and $\mathbf{b}$.

(f) $\mathbf{a} \times \mathbf{b}$

(g) $\mathrm{comp}_{\mathbf{a}}(\mathbf{v})$

(h) $\mathrm{proj}_{\mathbf{b}}(\mathbf{v})$

(i) $\mathbf{a} \cdot \mathbf{b} \times \mathbf{v}$

4. (5 points) Change the point $(4, 2, -4)$ to spherical coordinates.

5. (5 points) Find the parametric equations of the line through $(2, 5, 2)$ and parallel to the vector $\mathbf{v} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}$.

6. (10 points) Find the equation of the plane through the points $P = (2, 1, 3)$, $Q = (-2, 4, 7)$ and $R = (4, 1, -4)$?

7. (10 points) Where does the line through the points $(1, 2, 3)$ and $(4, -1, 1)$ intersect the plane $2x + 4y - 9z = 2$?

8. (10 points) What is the distance of the point $(1, 4, 3)$ from the plane $2x + y - 2z = 7$?

9. (15 points) An airplane is located at $(30, 40, 5)$ at noon and traveling with a velocity of $300\mathbf{i} + 400\mathbf{j} - \mathbf{k}$ kilometers per hour. The plot spots a landing strip at $(90, 120, 0)$

   (a) At what time is the plane directly over the landing strip?

   (b) How high is the plane when it is directly over the landing strip.

   (c) How long before the height of the plane above the ground is 2 kilometers?

10. An object moving in the direction $2\mathbf{i}+2\mathbf{j}+\mathbf{k}$ is acted on by a force given by the vector $\mathbf{i}+2\mathbf{j}+3\mathbf{k}$. Express this force as a sum of a force in the direction of motion and a force perpendicular to the direction of motion.