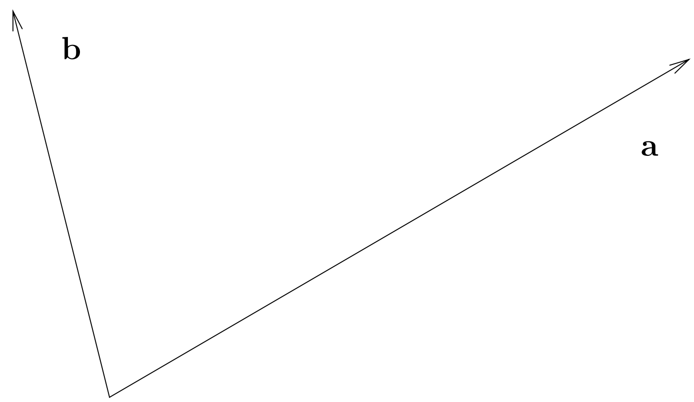
Mathematics 550 Final Name:______________________________

1. (10 points) In the figure draw and label both $\mathbf{a} - \mathbf{b}$ and $\mathrm{proj}_{\mathbf{a}}\, \mathbf{b}$



2. (20 points) Let $\mathbf{a} = -\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}$, $\mathbf{b} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$, $\mathbf{v} = \mathbf{i} - 2\mathbf{j} + \mathbf{k}$. Then compute

   (a) the angle between $\mathbf{a}$ and $\mathbf{b}$.

   (b) $\mathbf{a} \times \mathbf{b}$

   (c) $\mathrm{comp}_{\mathbf{a}}(\mathbf{v})$

   (d) $\mathrm{proj}_{\mathbf{b}}(\mathbf{v})$

3. (10 points) Find parametric equations of the line through the points $(1, 3, 4)$ and $(4, -3, 5)$

4. (10 points) Find the equation of the plane through the points $P = (2, 1, 3)$, $Q = (-2, 1, 6)$ and $R = (8, -4, 1)$?

5. (10 points) What is the distance of the point $(4, 5, 1)$ from the plane $2x + 2y - z = 7$?

6. (10 points) An object moving in the direction $\mathbf{v} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$ is acted on by a force given by the vector $\mathbf{F} = 2\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Express this force as a sum of a force $\mathbf{F}_{||}$ in the direction of motion and a force $\mathbf{F}_{\perp}$ perpendicular to the direction of motion.

$\mathbf{F}_{||} =$ ______________________________

$\mathbf{F}_{\perp} =$ ______________________________

7. (25 points) Complete the following identities:

(a) $\nabla(fg) =$

(b) $\operatorname{div}(f\mathbf{F}) =$

(c) $\operatorname{curl}(f\mathbf{F}) =$

(d) $\dfrac{d}{dt}\mathbf{b}(t)\cdot\mathbf{c}(t)$

(e) $\dfrac{d}{dt}\big(\mathbf{b}(t)\times\mathbf{c}(t)\big) =$

8. (10 points) What are the velocity, acceleration, and speed of the path $\mathbf{c}(t) = (t^2, \cos(t), \sin(t))$?

Velocity = ____________________

Acceleration= ____________________

speed= ____________________

9. (10 points) Sketch the graph of the curve parameterized by $x(t) = -2\cos(t)$ and $y(t) = 3\sin(t)$.

10. (10 points) What is the tangent plane to $x^2 + xy + yz + z^2 = 18$ at the point $(1, 2, 3)$.

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11. (10 points) What is the tangent line to $\mathbf{c}(t) = (t^2, t, t^3)$ when $t = 3$? ________________________

12. (10 points) Let $\mathbf{F} = \mathbf{i} + xz\mathbf{j} + \mathbf{k}xy^2$.

(a) Compute $\operatorname{curl} \mathbf{F}$. $\operatorname{curl} \mathbf{F} =$ ________________________

(b) Is there a function f so that $\mathbf{F} = \nabla f$? Explain your answer? HINT: What do you recall about $\operatorname{curl} \nabla f$?

13. (10 points) Let $\mathbf{c}\colon [a,b] \to \mathbf{R}^3$ be a curve so that $\mathbf{c}''(t) = -\|\mathbf{c}(t)\|^{-3}\mathbf{c}(t)$. Then show the vector

$$\mathbf{M}(t) = \mathbf{c}(t) \times \mathbf{c}'(t)$$

is constant.

14. (10 points) Let R be the region between the curves $y = x^2$ and $y = 2x + 3$. Then compute $\displaystyle\iint_R x^2 y \, dx \, dy$.

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15. (10 points) For the integral $\int_0^3 \int_{x^3}^{3x^2} f(x,y)\,dy\,dx$

(a) Draw the region of integration.

(b) Reverse the order of integration in the integral. ____________________

16. (10 points) Let B be the region bounded by $z=0$, $z=2$ and $x^2+y^2=1$. Then compute $\iiint_B \frac{z}{\sqrt{1+x^2+y^2}}\,dx\,dy\,dz$. HINT: Use cylindrical coordinates.

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17. (10 points) Set up (***but do not evaluate***) for the volume bounded by $z = x^2 + +4y^2$ and $z = 16$.

18. (10 points) Let $\mathbf{c}(t) = (t^2, 1+t, t^2)$ for $0 \le t \le 4$ and $\mathbf{F} = zy\mathbf{i} + xz\mathbf{i} + xy\mathbf{k}$. The compute $\int_{\mathbf{c}} \mathbf{F} \cdot d\mathbf{s}$.