Mathematics 527 Final Name: _____

Show your work to get credit. An answer with no work will not get credit.

(1) (20 points) Define the following:

(a) $x_k \to r$ ***linearly***.

(b) $x_k \to r$ ***quadratically***.

(c) r is a ***fixed point*** of $g(x)$.

(d) The ***divided difference*** $f[x_0, x_1, \ldots, x_n]$.

(e) The ***cardinal functions*** for the points $x_0, x_1, \ldots, x_n$.

(2) (20 points) State the following:

(a) State the n-th order Taylor theorem about x and with remainder for $f(x+h)$.

(b) Newton's method for solving the equations $f(x) = 0$.

(c) The trapezoid rule, with error term, for approximating $\int_a^b f(x)\,dx$.

(d) The error formula for approximating $f(x)$ by the polynomial of degree $\neq n$ that interpolates f at the points $x_0, \ldots, x_n$.

(3) (20 points) Let $f(x)$ be defined by

$$f(x) = \int_0^x \sin(t^2)\, dt$$

(a) Find the Taylor's expansion of $f(x)$ about the point $x = 0$.

(b) How many terms of this series are needed to compute $f(.1)$ to 10 decimal places?

(4) (15 points) Draw a graph $y = f(x)$ of some smooth function $f(x)$ and choice of initial point x_0 so that Newton's method for solving $f(x) = 0$ fails. (The equation $f(x) = 0$ should have at least one solution.)

(5) (15 points) If we have a sequence x_k from an application of Newton's method to find the root of r of $f(x) = 0$, so that the errors $e_k = r - x_k$ satisfy $|e_{k+1}| \leq (.01)e_k^2$ and the initial error $e_0 \leq 2$, then how many steps are needed to commute r accurate to 50 decimal places?

Number of steps = _____

(6) (15 points) A interpolating polynomial of degree 20 is used to approximate e^{-x} on the interval $[0, 2]$ at 21 equally spaced nodes. How accurate will this be?

(7) (15 points) Determine the error term in the approximation

$$f'(x) \approx \frac{8[f(x+h) - f(x-h)] - [f(x+h) - f(x-h)]}{12h}$$

(8) (15 points) Let $\phi(h)$ be a functions so that

$$\phi(h) = L + a_4 h^4 + a_6 h^6 + a_8 h^8 + \cdots$$

Then find a function ψ so that

$$\psi(h) = L + b_6 h^8 + b_8 h^8 + \cdots$$

for some constants b_8 and b_{10} and give the relationship between a_6, a_8 and b_6 and b_8.

(9) (15 points) How large must the even integer n be chosen in the composite Simpson's rule to insure that the error in computing $\int_0^1 \sin(x)\,dx$ is less than .00001?