

Math/Stat 511 Test #3

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (35 points) Let X be a random variable with the given function $M(t)$ as moment generating function. Then fill in the required information about X .

(a) $M(t) = (.4 + .6e^t)^4$

(i) What is the distribution of X ?

(ii) What is the pdf of X ?

(iii) What is $E(X)$?

$E(X) =$ _____

(iv) What is $P(x \geq 4)$?

$P(x \geq 4) =$ _____

(b) $M(t) = e^{2(e^t-1)}$

(i) What is the distribution of X ?

(ii) What is the pdf of X ?

(iii) What is the expect value of X ?

$\mu =$ _____

(iv) What is the variance of X ?

$\sigma^2 =$ _____

(c) $M(t) = e^{-7t+8t^2}$

(i) What is the distribution of X ?

(ii) What is the pdf of X ?

(iii) What is the expect value of X ?

$\mu =$ _____

(iv) What is the variance of X ?

$\sigma^2 =$ _____

(d) $M(t) = .2e^{2t} + .5e^{4t} + .3e^{6t}$.

(i) What is the pdf of X ?

(ii) What is $P(X = 6)$?

$P(X = 6) =$ _____

2. (10 points) The probability that a person has a side effect from a certain type of pain relief pill is .01. If 1000 people use this drug, then what is the probability that at most 8 people have the side effect?

3. (15 points) Let X be a random variable of continuous type with pdf

$$f(x) = \begin{cases} c(1+x) & -1 \leq x \leq 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of c $c =$ _____

(b) What is the expected value of X ? $E(X) =$ _____

(c) What is the distribution function of $f(x)$?

4. (10 points) Cars arrive at a toll booth at a mean rate of two a minute according to a Poisson distribution. What is the probability that the toll collector has to wait longer than 5 minutes to collect 12 tolls? **You can leave your answer as an integral.**

5. (15 points) Let X have a normal distribution with mean $\mu = 5$ and variance $\sigma^2 = 9$. Then find the following probabilities.

(a) $P(X \geq 5)$ $P(X \geq 5) =$ _____

(b) $P(X \leq 7.5)$ $P(X \leq 7.5) =$ _____

(c) $P(2 \leq X \leq 7)$ $P(2 \leq X \leq 7) =$ _____

6. (10 points) If X has the chi-square distribution $\chi^2(23)$ then find a and b so that $P(a < X < b) = 0.95$ and $P(X < a) = 0.025$.

$a =$ _____ $b =$ _____

7. (5 points) Let X be the value of a number chosen at random from the interval $3 \leq x \leq 12$. What is the probability that X is between 5 and 9.

$P(5 \leq X \leq 9) =$ _____