Math/Stat 511 Final Name:____________________________

Show your work! Answers that do not have a justification will receive no credit.

1. (15 Points)

   (a) Define what it means for events $A_1, \ldots, A_k$ to be mutually exclusive.

   (b) Define what if means for events A, B, and C to be independent.

   (c) Let A and B be events with $P(B) \neq 0$. Then what is the definition of the conditional probability $P(A|B)$?

   (d) Complete the following: **Probability** is a set function that assigns to each event A in the sample space S a number $P(A)$, called the probability of the event A, such that the following properties are satisfied:

   (e) Let X be a discrete random variable with space R. Then define the probability density function (p.d.f.) of X.

   (f) It X is a discrete random variable with space R and p.d.f. $f(x)$ then define the mathematical expectation $E(u(X))$ of $u(X)$.

2. (5 Points) If $S = B_1 \cup B_2$ with $B_1 \cap B_2 = \varnothing$ and A is an event with $P(A) \neq 0$ then give the derivation of Bayes' Law for $P(B_2|A)$. (It is fine if you do this in terms of a tree diagram.)

3. (10 Points) Let A and B be events so that $P(A) = .5$, $P(B) = .7$, $P(A \cup B) = .8$. Then compute

(a) $P(A')$ $\qquad P(A') =$ ________________

(b) $P(A \cap B)$ $\qquad P(A \cap B) =$ ________________

(c) $P(A \cup B')$ $\qquad P(A \cup B') =$ ________________

4. (5 Points) Assume the events A and B are independent and that $P(A) = .5$ and $P(B) = .2$. Then what is $P(A \cup B)$?

$P(A \cup B) =$ ________________

5. (5 Points) Alice and Bill play a checker tournament where the first person to win 5 games wins the tournament. How many different ways can Alice win the tournament in exactly 8 games?

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6. (10 Points) Let A_1 and A_2 be the events that that a person is left or right handed, respectively. Let B_1 and B_2 be the events that a person is left eye dominant or right eye dominant, respectively. A survey in one statistics class yielded the following data:

	B_1	B_2	Total
A_1	20	15	35
A_2	30	25	55
Total	50	40	90

Compute the following:

(a) $P(A_2)$

$P(A_2) =$ ____________________

(b) $P(B_2|A_1)$

$P(B_2|A_1) =$ ____________________

(c) The probability that a member of the class is left handed given that they are right eye dominant.

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7. (10 Points) Assuming that it is equally likely that a person be born on any of the 7 days of the week, then what is the probability that of 7 people chosen at random no two were born in the same day of the week?

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8. (10 Points) Two cards are drawn at random without replacement from a standard deck of 52 playing cards. Then compute

(a) The probability that one is a club and the other is a heart. ______________________

(b) The first one is a King and the second one is Black. ______________________

9. (10 Points) Students coming form high school district A have a 70% chance of passing freshman calculus, while students coming from high school district B have a 80% chance of passing the class. If a freshman calculus class has 25% of its students from district A and the remaining 75% from district B, then what is the probability that a student who passes the class can from district A?

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10. (10 Points) Let X be a discrete random variable with p.d.f.

$$f(x) = \frac{3+x}{6}, \qquad x = -2, -1, 0.$$

Find the mean and variance of X. $\mu =$ ________________

$\sigma^2 =$ ________________

11. (5 Points) A box of 12 donuts has 8 plain and 4 chocolate donuts. If 6 donuts are choosen at random then what is the probablity that exactly 2 are chocloate?

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12. (10 Points) If 20% of the students at U.S.C. are left handed, let X be the number of left handed people out of a random sample of 20 students.

(a) What is the expected number of people in the sample that are left handed. ________________

(b) Compute $P(3 \leq X \leq 6)$ $P(3 \leq X \leq 6) =$ ________________

13. (20 points) Let X be a random variable with the given function $M(t)$ as moment generating function. Then fill in the required information about X.

(a) $M(t) = (.7 + .3e^t)^5$

(i) What is the distribution of X?

(ii) What is the pdf of X?

(iii) What is $E(X)$? $E(X) =$ ____________

(iv) What is $P(X = 3)$? $P(X = 3) =$ ____________

(b) $M(t) = e^{-6t+24t^2}$

(i) What is the distribution of X?

(ii) What is the pdf of X?

(iii) What is the expect value of X? $\mu =$ ____________

(iv) What is the variance of X? $\sigma^2 =$ ____________

(c) $M(t) = .1e^{-2t} + .7e^t + .2e^{3t}$.

(i) What is the pdf of X?

(ii) What is $P(X = 6)$? $P(X = 6) =$ ____________

14. (10 points) Let X be a random variable of continuous type with pdf

$$f(x) = \begin{cases} cx(1-x) & 0 \leq x \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Find the value of $c =$ ______________________

(b) What is the expect value of X? $E(X) =$ ______________________

(c) What is the distribution function of X?

15. (5 points) Let X have a normal distribution with mean $\mu = 3$ and variance $\sigma^2 = 4$. Then find the following probabilities. Then find the probablity $P(1 \leq X \leq 7)$

$P(1 \leq X \leq 7) =$ ______________________

16. (10 Points) Let X have the p.d.f. $f(x) = 5x^4$ for $0 \leq x \leq 1$. Then find the p.d.f. of $Y = X^{1/3}$.