

Test 3

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1.(30 points) Find a particular solution to each of the following:

(a) $y'' - 8y' + 25y = 4 \cos(3x)$

(b) $y'' + 4y' - 8y = 3x^2 - 4x + 9$

(c) $y'' - y' - 2y = 3 + 4e^{2x}$

2. (15 points) The functions $y_1 = x$ and $y_2 = x^2$ are solutions to $x^2y'' - 2y' + 2y = 0$. Find the general solution to $x^2y'' - 2y' + 2y = \frac{3}{x}$

3. (15 points) The function $y_1 = x$ is a solution to $5x^2y'' + xy' - y = 0$. Find the general solution to this equation.

4. (10 points) Let $L[y] = y'' + py' + qy$. Then show that $L[c_1y_1 + c_2y_2] = c_1L[y_1] + c_2L[y_2]$.

5. (15 points) A mass of weighting 12 lbs is attached both a vertically suspended spring with a spring constant of $k = 6$ and a dashpot that provides 3 lbs of resistance for every foot per second of velocity (that is the dashpot constant is $c = 3$). If the mass is pulled down 1 ft below its static equilibrium position and released from rest,

(a) Find a formula for the position of the mass after t seconds. (Use $g = 32\text{ft}/\text{sec}^2$).

6. (15 points). A mass weighing 100 lb is attached to the end of a spring with spring constant $k = .01$. A force of $F = F_0 \cos(\omega t)$ acts on the mass. At what value of ω will resonance occur? (That is what is the value of ω leads to unbounded solutions?) (Note it may be possible to answer this question without find the complete solution to the equation.)