

Test 1

Name: _____

Show your work! Answers that do not have a justification will receive no credit.

1. (30 points) (a) Solve the initial value problem $y''(t) = 3 + 12t$, $y(0) = 4$, $y'(0) = 1$

(b) $y' = 4y$, $y(0) = -3$

(c) Find the general solution to $y' = \frac{e^{2x+y}}{y}$

(d) $x^2y' + 3xy = 4x^2$, $y(1) = 7$.

2. (10 points) What values of the constant r make $y = e^{rx}$ a solution to $y'' - 5y' + 4y = 0$?

3.(20 points) Recall that by Newton's law of cooling an object cools at a rate proportional to the difference between its temperature and the temperature of the surrounding air. A cup of water cooled to 40°F is set in a room that is 80°F . Its temperature 20 minutes later is 50°F .

(a) Find a formula for the temperature of the cup after t minutes.

(b) How long does it take for the cup to be 70°F ?

4. (20 points) The starling is not native to the United States, but was introduced to this country when 24 starlings were released into central park in the early 1800's. Five year after their release there where 240 of them. Assuming that the rate of increase of the population of starlings was proportional to the number in the population, how long after the initial release before there were 1,000,000 starlings in the United States?

5. (20 points) A tank holds 100L of water when it is full. At an initial time the tank contains 50L of water that has 10Kg of salt dissolved in it. At this time water that has 2Kg per L of salt dissolved in it is pumped in at the rate of 3L/min, and same time water is drained out at 2L/min.

(a) How long before the tank is full?

(b) How much salt is in the tank when it becomes full?