Mathematics 174 Test #2   Name: _____

Show your work to get credit. An answer with no work will not get credit.

(1) (20 Points)
  (a) State the quotient remainder theorem.

  (b) Define *precisely* the following:
     (i) n div d

     (ii) $n \mod d$

     (iii) $\lfloor x \rfloor$

     (iv) $\lceil x \rceil$

(2) (10 Points) Compute the following:
  (a) 45 div 13 $=$

  (b) $-29 \mod 12 =$

  (c) $\lfloor 39/8 \rfloor =$

  (d) $\lceil -75/32 \rceil =$

(3) (5 Points) Show that $\sqrt{7}$ is irrational. You many use the following:
   Proposition: *If n is an integer and $7 \mid n^2$, then $7 \mid n$.*

(4) (5 Points) Show that if $n \mod 6 = 4$ then $\left\lceil \dfrac{n}{6} \right\rceil = \dfrac{n+2}{6}$

(5) (5 Points) Show that if the sum of three numbers is greater then 30, then at least one of the numbers is greater than 10.

(6) (10 Points) For the following either give a proof or a counterexample.
   (a) The difference of two irrational numbers is irrational.

   (b) The difference of a rational and an irrational number is irrational.

(7) (10 Points) Compute the following

(a) $\displaystyle\sum_{k=2}^{5}(2k^2+3)$ _____

(b) $\displaystyle\prod_{m=1}^{5}\frac{2m-1}{2m+1}$ _____

(c) $\displaystyle\sum_{k=1}^{20}\left(\frac{2}{k}-\frac{2}{k+1}\right)$

(8) (5 Points) What is a formula for the general term a_k of the sequence that starts
$$\frac{3}{2},\ \frac{-5}{4},\ \frac{7}{6},\ \frac{-9}{8},\ \frac{11}{10},\cdots$$

(9) (10 Points) Write the following using summation or product notation.

(a) $3^2-4^2+5^2-6^2+7^2-8^2+9^2-10^2+11^2-12^2$ _____

(b) $1\cdot 3\cdot 5\cdot 7\cdot 9\cdot 11\cdot 13\cdot 15$. _____

(c) $3+5+7+\cdots+2n+1$ _____

(10) (5 Points) Use induction to show that $9^n - 1$ is divisible by 8 for all $n \geq 1$.

(11) (10 Points) Use induction to show that the sum of the first n odd numbers is n^2. In symbols this means to show that

$$1 + 3 + 5 + \cdots + 2n - 1 = n^2.$$