Mathematics 172 Homework

1. A agriculturist is feeding a pond full of tilapia by growing duckweed in the pond. Let $A(t)$ be the number of kg of duckweed in the pond t weeks. Because of the feeding of the fish the intrinsic growth rate of the duckweed of the duckweed is $r = -.15$.

(a) What is the rate equation satisfied by $A(0)$ if no duckweed is stocked in the pond.

Answer: $\dfrac{dA}{dt} = -.15A$.

(b) What are the units of r? *Answer:* $(kg/week)/kg$.

(c) The agriculturist stocks starts to stock the pond with duckweed at a constant rate of 5 kg/week. What is the new rate equation satisfied by $A(t)$?

Answer: $\dfrac{dA}{dt} = -.15A + 5$.

(d) What is the equilibrium solution to this equation.

Answer: $A = 5/.15 = 33.33$

(e) Draw graphs of the solutions to the rate equation with the initial values $A(0) = 25$, $A(0) = 30$, $A(0) = 40$.

(f) For the solution you graphed the last part, estimate $A(52)$.

Answer: $A(52) \approx 33.33$ for all these solution.

(g) Find the exact solution with initial value $A(0) = 25$. What is is the exact value of $A(52)$. What is the value of $A(20)$?

Answer: $A(t) = 33.33 - 8.33e^{-.15t}$, $A(52) = 33.327$, and $A(20) = 32.915$

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2. Due to fishing pressure the number of bass in a lake has a negative intrinsic growth rate of $r = -.2$. Let the number of bass in the lake be $N(t)$ where t is in years.

(a) What is the rate equation satisfied by $N(t)$?

Answer: $\dfrac{dN}{dt} = -.2N$.

(b) What are the units of r? *Answer:* (fish/year)/fish.

(c) DNR (Department of Natural Resources) wants the lake to have a stable population of 40,000 bass. At what rate should they stock the lake?

Answer: Let S fish/year be the rate at which the lake is stocked . Then the rate equation satisfied by N becomes

$$\frac{dN}{dt} = -.2N + S.$$

The equilibrium solution for this equation is found by solving $-.2N + S$ for N. This gives

$$N = \frac{S}{.2}.$$

This will be the stable population size of the lake. We want this to be

$$\frac{S}{.2} = 40{,}000$$

which gives

$$S = (.2)(40{,}000) = 8{,}000\text{fish/year}$$

as the required stocking rate.