Mathematics 172 Homework

1. Consider the the rate equation

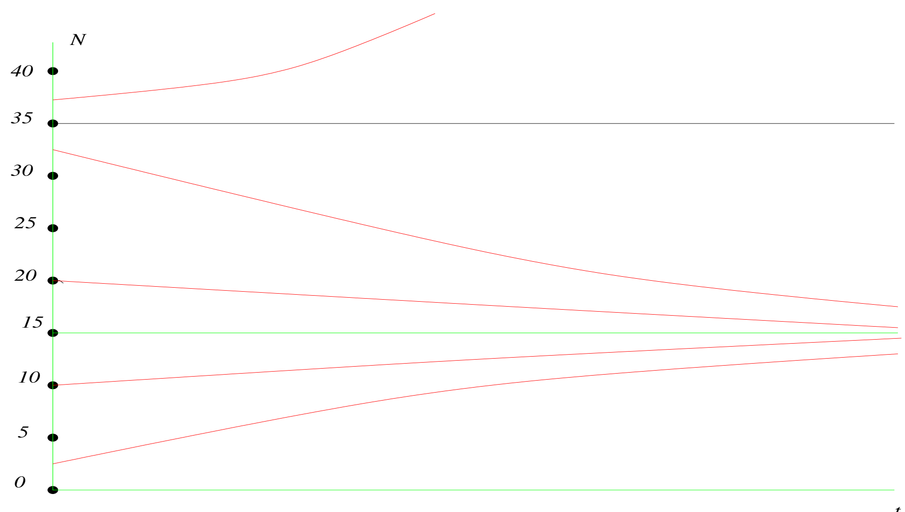
$$\frac{dN}{dt} = .13N(n-15)(n-35)$$

(a) What are the ***equilibrium solutions***. That is the solutions that are constants.

Answer: Set $\dfrac{dN}{dt} = .13N(n-15)(n-35) = 0$ and get $N = 0$, $N = 15$ and $N = 35$ as the equilibrium solution.

(b) Sketch the graphs of the solutions with the following initial values $N(0) = 2.5$, $N(0) = 10$, $N(0) = 20$, $N(0) = 32.5$, $N(0) = 37.5$

Answer: The equilibrium solutions are in green and the other required solutions are in red.

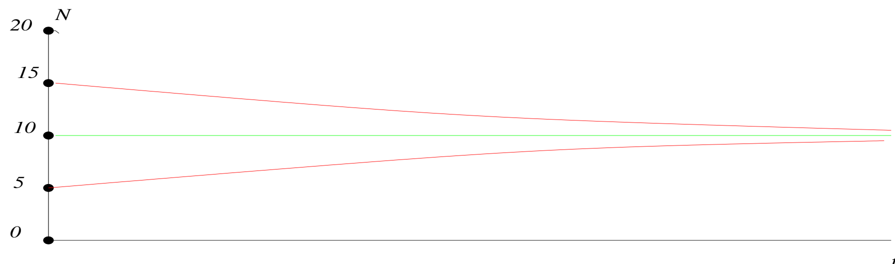


2. This time consider the rate equation

$$\frac{dP}{dt} = -2(P-10).$$

(a) Find the equilibrium solution. *Answer:* $P = 10$.

(b) Graph the equilibrium solution along with the solutions with initial values $P(0) = 5$ and $P(0) = 15$



(c) The figure makes it look like the solution could be exponential decay towards $P = 10$. So see if this is the case do a change of variable,

$$y = P - 10.$$

Show that y satisfies the rate equation

$$\frac{dy}{dt} = -2y.$$

Hint: $\dfrac{dy}{dt} = \dfrac{dP}{dt}$ as the derivative of 10 is zero.

(d) Show that y is given by

$$y(0) = y_0 e^{-2t}.$$

(e) So if we have the solution $P(t)$ of the original equation with $P(0) = 5$, then $y(0) = P(0) - 10 = 5 - 10 = -5$. Therefore

$$y(0) = y_0 e^{-2t} = -5e^{-2t}.$$

But $y = P - 10$ is implies $P = y + 10$. Therefore

$$P(t) = 10 - 5e^{-2t}.$$

Use the same change of variable $y = P - 10$ to show that the solution to $\frac{dP}{dt} = -2(P - 10)$ and $P(0) = 10$ is

$$P(t) = 5 + 5e^{-2t}.$$

3. Use the ideas of the last problem to find the solutions of

$$\frac{dN}{dt} = .5(N - 50)$$

with initial conditions $P(0) = 40$ and $P(0) = 65$. *Answer:* This time the change of variable is $y = N - 50$ and the equation for y is $y' = .5y$.

Answer: The solution for $N(0) = 40$ is $N(t) = 50 - 10e^{.5t}$ and the solution for $N(0) = 65$ is $N(t) = 50 + 15e^{.5t}$.