Mathematics 172 Homework

1. Duckweed (a small floating plant) is growing in a pond. As some point a chemical starts to dump toxins into the pond. After two weeks only 90% of the duckweed is left. Assume a growth rate of that satisfies the rate equation $N' = rN$,

(a) Find the per capita growth rate r.

Answer: The growth is exponential, $N = N_0 e^{rt}$. After 2 weeks 90% is left, so $N(2) = N_0 e^{2r} = .9N_0$. This gives the equation $e^{2r} = .9$, whcih we can solve for r to get $r = \ln(.9)/2 = -0.05268$

(b) Find a formula fraction of the duckweed that is left t week after the dumping starts.

Answer: Using our value of r we have $N = N_0 e^{-0.05268t}$. So the fraction of the original amount is $e^{-0.05268t}$.

(c) How long until only 10% of the duckweed is left.

Answer: We need to solve $e^{-0.05268t} = .1$. This gives $t = \ln(.1)/(-0.05268) = 43.71$ weeks.

2. One hundred guppies (a small fast breeding fish) are introduced to an abandoned swimming pool to control mosquitoes. A month and a half later there are 750 guppies in the pool. Without constraints these fish grow according to the rate equation $N' = rN$ where N is the population size.

(a) Find the per capita growth rate r.

Answer: If $N(t)$ is the number of guppies after t months, then $N = N_0 e^{rt} = 100e^{rt} = 100e^{rt}$. Then $N(1.5) = 100e^{1.5r} = 750$. This leads to $r = \ln(750/100)/1.5 = 1.343$.

(b) Find a formula for the number of guppies in the pool t months after the original one hundred are released.

Answer: $N(t) = 100e^{1.343t}$

(c) How long until there are one hundred thousand guppies in the pond?

Answer: Solve $N(t) = 100e^{1.343t} = 100000$, to get $t = \ln(100000/100)/1.343 = 5.143$ months.