Mathematics 172 Homework

Islands off the cost of New Guinea are the homes of a species of fruit bat. The probability of an island having its population of bats going extinct is $p_e = .5$. The probability of a island with no bats being colonized by bats from a nearby island is $p_i = .8$ Let f be the proportion, or fraction, of the islands that are populated by the bats.

1. Write down the rate equation satisfied by f. Answer:

$$\frac{df}{dt} = p_i(1-f) - p_e f = -(p_i + p_e)f + p_i = -(.5+.8)f + .8 = -1.3f + .8$$

2. What is equilibrium point for this equation? Answer: $\hat{f} = .8/.13 = 8/13 = .61538$

3. If f(0) = .3 estimate f(100). Answer: $f(100) \approx .61538$

4. If f(0) = .3 find the exact solution of the initial value problem

$$\frac{df}{dt} = -1.3f + .8$$

and use it to find f(2), f(5) and f(20). Answer: $f(x) = .61538 + (.3 - .61538)e^{-1.3t} = .61538 - .31538e^{-1.3t}$, f(2) = .59195, f(5) = .61491, f(20) = .61538.