Mathematics 172 Homework

In a discrete dynamical system $N_{t+1} = f(N_t)$ the ***equilibrium points*** (also called a ***stationary point*** or a ***rest point***), N_*, are the points solutions to $f(N) = N$. If $|f'(N_*)| < 1$, then it is a ***stable*** (or ***attracting***) equilibrium points. If $|f'(N_*)| > 1$ it is ***unstable*** (or ***repelling***).

In the special case of the discrete logistic equation with carrying capacity K and per capita grow rate r, that is

$$N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right) \tag{1}$$

there will be two equilibrium points, $N = 0$ and $N = K$.

- The equilibrium at $N = 0$ is always unstable.
- The equilibrium at $N = K$ is always unstable is stable when $0 < r < 2$ and unstable when $r > 2$.

Note that the right hand side of this equation also has a zero when $N_t = \dfrac{1+r}{r}K$.

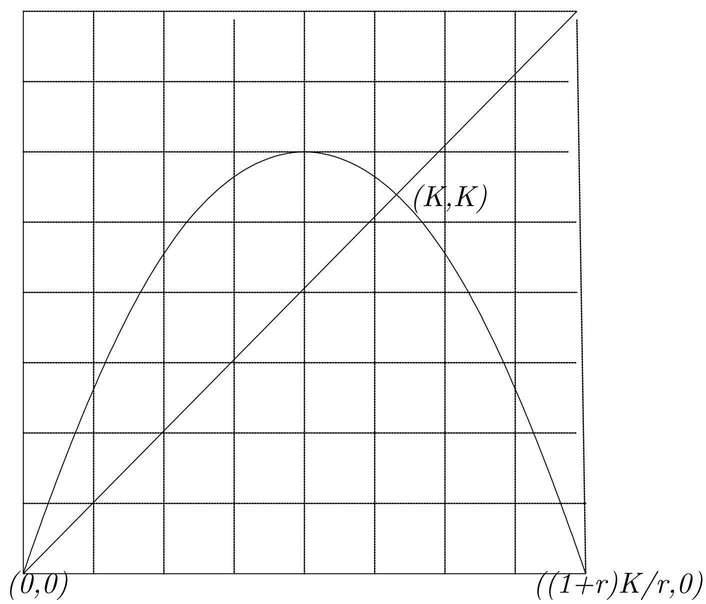

FIGURE 1. The graph of $N_{t+1} = N_t + rN_t\left(1 - \frac{N_t}{K}\right)$ as a function of N_t, showing the two equilibrium points $(0,0)$ and (K,K). Also showing that the zeros of the of the right hand side of (1) are $N = 0$ and $N = \dfrac{1+r}{r}K$.

1. For a discrete logistic growth with $r = .65$ and $K = 200$ and an initial population size of $N_0 = 150$.

(a) Compute N_1, N_2, N_3, and N_4 *Answer:* $N_1 = 174.3750$, $N_2 = 188.8971$, $N_3 = 195.71337$, and $N_4 = 198.4399$. Note in the case $r < 2$ so the carrying capacity $K = 200$ is a stable rest point. And the points are getting closer and closer to $K = 200$.

(b) If we had started with $N_0 = 235$, estimate N_{100}. *Answer:* As $K = 200$ is stable we have $N_{100} \approx 200$.

2. For a discrete logistic growth with $r = 2.5$ and $K = 200$ and an initial population size of $N_0 = 150$.

(a) Compute N_1, N_2, N_3, and N_4 *Answer:* $N_1 = 243.7500$, $N_2 = 110.4492$, $N_3 = 234.0843$, and $N_4 = 134.3515$. This time $r > 2$ so the carrying capacity is unstable. Thus the values of N_t will jump around and we will not get get closer and closer to $K = 200$.

3. The discrete logistic growth has the flaw that for large values of N_t it becomes negative and we can not have a negative number of animals. A model that does not have this problem is

$$N_{t+1} = rN_te^{-N_t/K}$$

In the case $r = 1.2$ and $K = 100$ use your calculator to find the equilibrium points and determine if they are stable or unstable. *Answer:* $N_* = 0$ and $N_* = 100$. Here $N_* = 0$ is unstable and $N_* = 100$ is stable.

Here are some graphs for discrete dynamical systems. Answer the questions about the equilibrium point. The answers are on page 6.

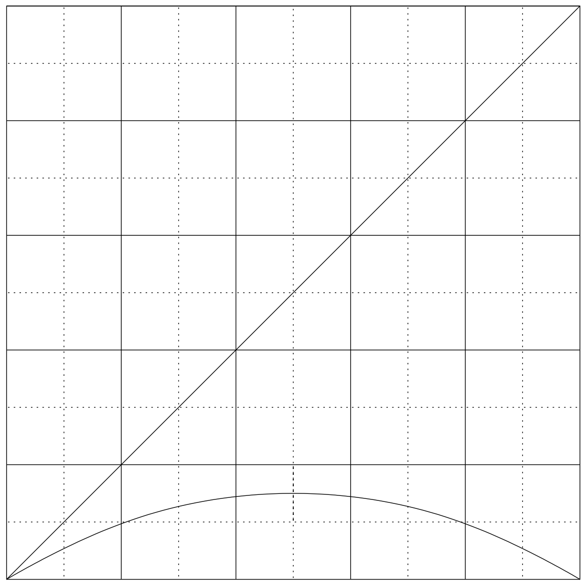

FIGURE 2. In this system there is only one equilibrium point, the one at $N = 0$. Is is stable or unstable? Draw some cobwebs to decide.

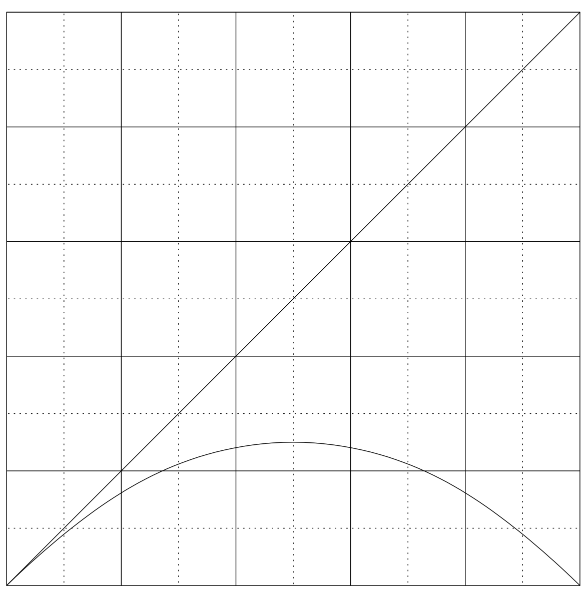

FIGURE 3. Again there is only one equilibrium point, the one at $N = 0$. Is is stable or unstable? Draw some cobwebs to decide.

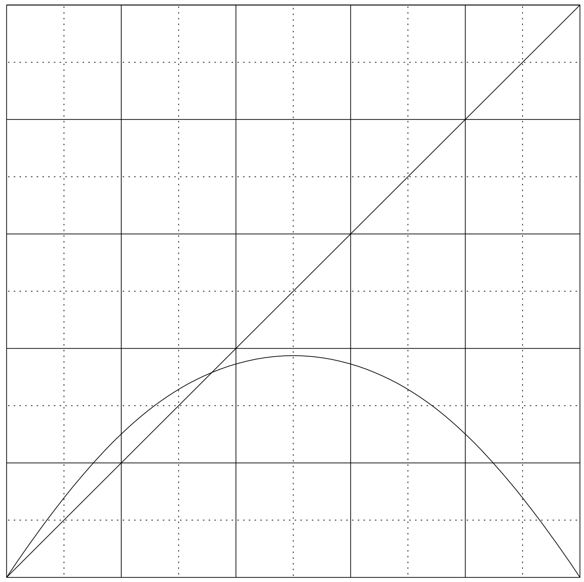

FIGURE 4. Here there are two equilibrium points. Are they stable or unstable?

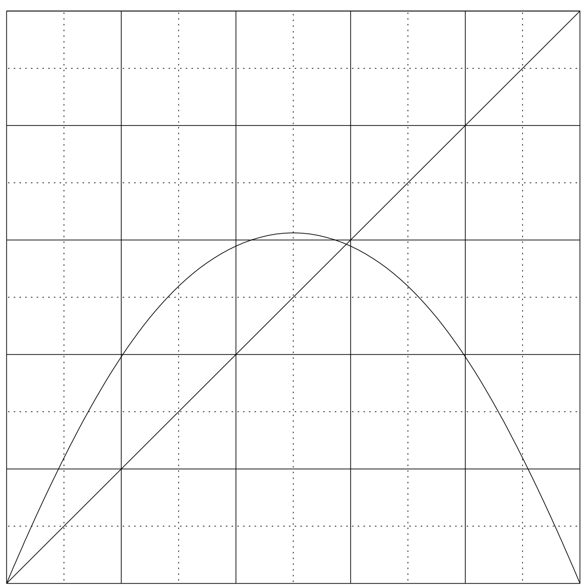

FIGURE 5. Again decide if the two equilibrium points are stable or unstable.

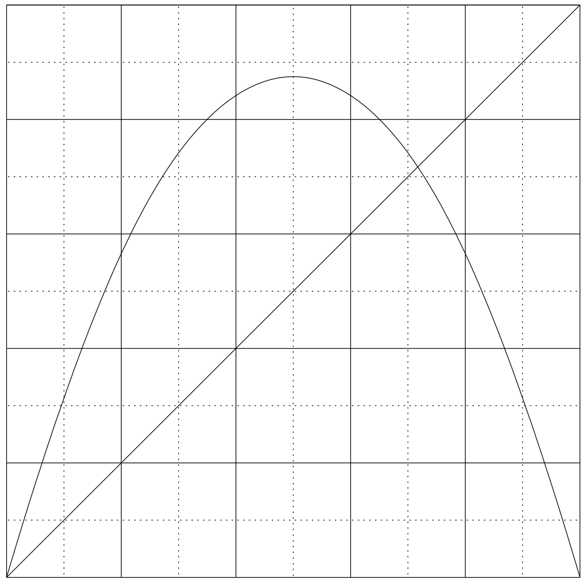

FIGURE 6. Are the two equilibrium points are stable or unstable.

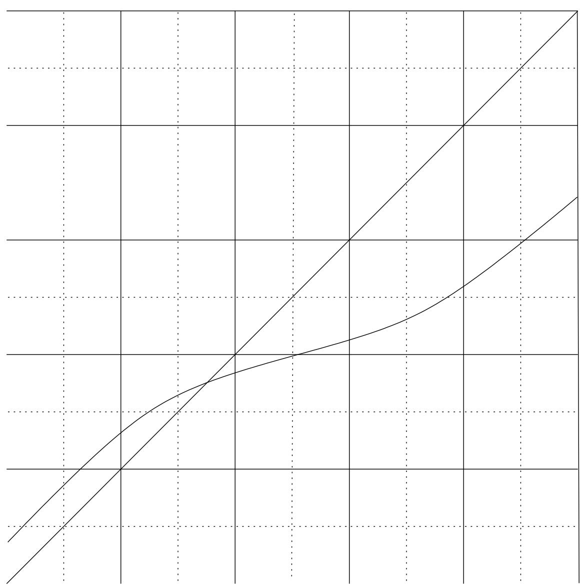

FIGURE 7. Is the equilibrium point stable or unstable?

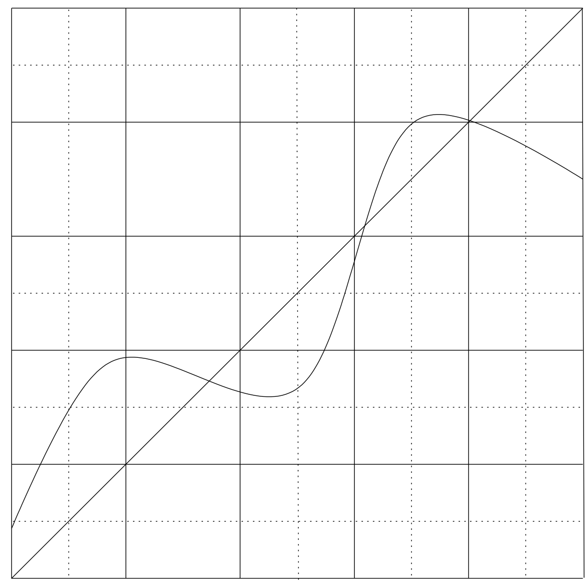

FIGURE 8. Here there are three equilibrium points. Which are stable and which unstable?

Answers:
Figure 2: 0 is stable.
Figure 3: 0 is stable.
Figure 4: 0 is unstable and the other rest point is stable.
Figure 5: 0 is unstable and the other rest point is stable.
Figure 6: Both points are unstable.
Figure 7: The point is stable.
Figure 8: The middle rest point is unstable, the other two are stable.