## Mathematics 172 Homework

Let us get back to the logistic equation equation

$$
\frac{d P}{d t}=r P\left(1-\frac{P}{K}\right)
$$

where $r$ is the intrinsic growth rate and $K$ is carrying capacity. We know do some variants on this equation.

1. Let $P(t)$ be the number of grams of algae in an aquarium $t$ months after it is set up. Originally the algae grows logistically with $r=.15$ and $K=15$.
(a) What is the differential equation satisfied by $P$ ?

Answer: $\frac{d P}{d t}=.15 P\left(1-\frac{P}{15}\right)$.
A some point algae eating snails are introduced to the aquarium. They eat the algae are a rate of $10 \%$ of the amount present.
(b) What is the new differential equation satisfied by $P$ ?

Answer: $\frac{d P}{d t}=.15 P\left(1-\frac{P}{15}\right)-.1 P$
(c) What is the stable population size of the algae after the snails are introduced?

Answer: $P=5$.
(d) If the snails eat $20 \%$ of the amount of algae present, what is the new rate equation and stable population size?

Answer: $\frac{d P}{d t}=.15 P\left(1-\frac{P}{15}\right)-.2 P$ and $P=0$ (that is the algae dies out).
2. With the same logistic rate equation for the amount of algae in the aquarium, that is

$$
\frac{d P}{d t}=.15 P\left(1-\frac{P}{15}\right)
$$

This time a filter is put in that removes the algae at a continuous rate of .2 grams/month.
(a) What is the new rate equation for $P$ ?

Answer: $\frac{d P}{d t}=.15 P\left(1-\frac{P}{15}\right)-.2$
(b) What are the equilibrium points of this equations?

Answer: We need to solve $.15 P\left(1-\frac{P}{15}\right)-.2=0$. This is not easy to do by hand, so we use the calculator. Use the $Y=$ button and enter $\backslash Y 1=.15 * X *(1-X / 15)-.2$ Use the WINDOW button and $\mathrm{Xmin}=0$ and $\mathrm{Xmax}=15$. Then do a ZoomFit to plot the function. You should get something that looks like:


Now use 2nd calc and then the zero function to find that the two points where the graph crosses the $x$ axis are $P=1.479$ and $P=13.521$. These are the equilibrium points.
(c) Which of these two points is stable? Answer: $P=13.521$

