## Mathematics 172 Homework

Let us get back to the logistic equation equation

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right)$$

where r is the intrinsic growth rate and K is carrying capacity. We know do some variants on this equation.

1. Let P(t) be the number of grams of algae in an aquarium t months after it is set up. Originally the algae grows logistically with r = .15 and K = 15. (a) What is the differential equation satisfied by P?

Answer:  $\frac{dP}{dt} = .15P\left(1 - \frac{P}{15}\right)$ .

A some point algae eating snails are introduced to the aquarium. They eat the algae are a rate of 10% of the amount present.

(b) What is the new differential equation satisfied by P?

Answer:  $\frac{dP}{dt} = .15P\left(1 - \frac{P}{15}\right) - .1P$ 

(c) What is the stable population size of the algae after the snails are introduced?

Answer: P = 5.

(d) If the snails eat 20% of the amount of algae present, what is the new rate equation and stable population size?

Answer:  $\frac{dP}{dt} = .15P\left(1 - \frac{P}{15}\right) - .2P$  and P = 0 (that is the algae dies out).

2. With the same logistic rate equation for the amount of algae in the aquarium, that is

$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{15}\right).$$

This time a filter is put in that removes the algae at a continuous rate of .2 grams/month.

P?

(a) What is the new rate equation for  
Answer: 
$$\frac{dP}{dt} = .15P\left(1 - \frac{P}{15}\right) - .2$$

(b) What are the equilibrium points of this equations?

Answer: We need to solve  $.15P\left(1-\frac{P}{15}\right) - .2 = 0$ . This is not easy to do by hand, so we use the calculator. Use the Y= button and enter Y1 = .15\*X\*(1-X/15)-.2 Use the WINDOW button and Xmin=0 and Xmax=15. Then do a ZoomFit to plot the function. You should get something that looks like:



Now use 2ND CALC and then the zero function to find that the two points where the graph crosses the x axis are P = 1.479 and P = 13.521. These are the equilibrium points.

(c) Which of these two points is stable? Answer: P = 13.521