## Mathematics 172 Homework

Example 1. A cell has volume $V=8 \times 10^{-6} \mathrm{~mm}^{3}$ and surface area $A=$ $3.6 \times 10^{-3} \mathrm{~mm}^{2}$. Assume that oxygen, $O_{2}$, passes through the cell membrane at a rate of $.5\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}$
(a) What is the total ammount of $O_{2}$ that is comming into the cell per hour?

## Solution:

Total $O_{2} /$ hour $=\left(3.6 \times 10^{-3} \mathrm{~mm}^{2}\right) \times .5\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}=.0018 \mathrm{mg} / \mathrm{hr}$.
(b) What is the amount of $O_{2}$ per volume comming into the cell per hour?

Solution: Take the last answer and divide by the volume:

$$
\text { Rate of } O_{2} \text { per volume }=\frac{.0018 \mathrm{mg} / \mathrm{hr}}{8 \times 10^{-6} \mathrm{~mm}^{3}}=225\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr} .
$$

(c) If the cell needs $50\left(\mathrm{mg} / \mathrm{mm}^{3}\right) / \mathrm{hr}$ of $O_{2}$ to survive, then how much can it be magnified before it dies from lack of oxygen?

Solution: Let $a$ be the factor by which it is magnified. Then by our rules for scaling we have we have

$$
V_{\text {mag }}=8 \times 10^{-6} a^{3} \mathrm{~mm}^{3}, \quad A_{\text {mag }}=3.6 \times 10^{-3} a^{2} \mathrm{~mm}^{2}
$$

Thus

$$
\text { Total } O_{2} \text { intake }=A_{\operatorname{mag}} \times .5\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}=.0018 a^{2} \mathrm{mg} / \mathrm{hr}
$$

and

$$
\text { Rate of } O_{2} \text { per volume }=\frac{.0018 a^{2} \mathrm{mg} / \mathrm{hr}}{8 \times 10^{-6} a^{3} \mathrm{~mm}^{3}}=\frac{225\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}}{a}
$$

The threshold where oxygen starvation sets in is when

$$
\text { Rate of } O_{2} \text { per volume }=50\left(\mathrm{mg} / \mathrm{mm}^{3}\right) / \mathrm{hr} .
$$

That is

$$
\frac{225\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}}{a}=50\left(\mathrm{mg} / \mathrm{mm}^{3}\right) / \mathrm{hr} .
$$

Solving for $a$ gives

$$
a=\frac{225}{50}=4.5
$$

Therefore the cell can only grow to 4.5 times its length.

1. A cell has volume $V=4.6 \times 10^{-6} \mathrm{~mm}^{3}$ and surface area $A=6.7 \times$ $10^{-3} \mathrm{~mm}^{2}$. Assume that oxygen, $O_{2}$, passes through the cell membrane at a rate of $.62\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}$
(a) What is the total ammount of $O_{2}$ that is comming into the cell per hour? Answer: $4.154 \times 10^{-3} \mathrm{mg} / \mathrm{hr}$.
(b) What is the amount of $O_{2}$ per volume comming into the cell per hour? Answer: $903.04\left(\mathrm{mg} / \mathrm{mm}^{2}\right) / \mathrm{hr}$.
(c) If the cell needs $377\left(\mathrm{mg} / \mathrm{mm}^{3}\right) / \mathrm{hr}$ of $O_{2}$ to survive, then how much can it be magnified before it dies from lack of oxygen?

Answer: The manification factor is $a=18.06$.

