Mathematics 172 Homework

Example 1. A cell has volume $V = 8 \times 10^{-6} \text{mm}^3$ and surface area $A = 3.6 \times 10^{-3} \text{mm}^2$. Assume that oxygen, O_2 , passes through the cell membrane at a rate of $.5(\text{mg/mm}^2)/\text{hr}$

(a) What is the total ammount of O_2 that is comming into the cell per hour?

Solution:

Total O_2 /hour = $(3.6 \times 10^{-3} \text{mm}^2) \times .5 (\text{mg/mm}^2)/\text{hr} = .0018 \text{mg/hr}.$

(b) What is the amount of O_2 per volume comming into the cell per hour? Solution: Take the last answer and divide by the volume:

Rate of
$$O_2$$
 per volume = $\frac{.0018 \text{mg/hr}}{8 \times 10^{-6} \text{mm}^3} = 225 (\text{mg/mm}^2)/\text{hr}.$

(c) If the cell needs $50(\text{mg/mm}^3)/\text{hr}$ of O_2 to survive, then how much can it be magnified before it dies from lack of oxygen?

Solution: Let a be the factor by which it is magnified. Then by our rules for scaling we have

$$V_{mag} = 8 \times 10^{-6} a^3 \text{mm}^3, \qquad A_{mag} = 3.6 \times 10^{-3} a^2 \text{mm}^2$$

Thus

Total
$$O_2$$
 intake = $A_{mag} \times .5 (\text{mg/mm}^2)/\text{hr} = .0018a^2 \text{mg/hr}$

and

Rate of
$$O_2$$
 per volume = $\frac{.0018a^2 \text{mg/hr}}{8 \times 10^{-6}a^3 \text{mm}^3} = \frac{225(\text{mg/mm}^2)/\text{hr}}{a}$.

The threshold where oxygen starvation sets in is when

Rate of
$$O_2$$
 per volume = $50 (\text{mg/mm}^3)/\text{hr}$.

That is

$$\frac{225(\text{mg/mm}^2)/\text{hr}}{a} = 50(\text{mg/mm}^3)/\text{hr}.$$

Solving for a gives

$$a = \frac{225}{50} = 4.5$$

Therefore the cell can only grow to 4.5 times its length.

1. A cell has volume $V = 4.6 \times 10^{-6} \text{mm}^3$ and surface area $A = 6.7 \times 10^{-3} \text{mm}^2$. Assume that oxygen, O_2 , passes through the cell membrane at a rate of $.62(\text{mg/mm}^2)/\text{hr}$

(a) What is the total ammount of O_2 that is comming into the cell per hour? Answer: 4.154×10^{-3} mg/hr.

(b) What is the amount of O_2 per volume comming into the cell per hour? Answer: $903.04(\text{mg/mm}^2)/\text{hr}$.

(c) If the cell needs $377 (mg/mm^3)/hr$ of O_2 to survive, then how much can it be magnified before it dies from lack of oxygen? Answer: The manification factor is a = 18.06.