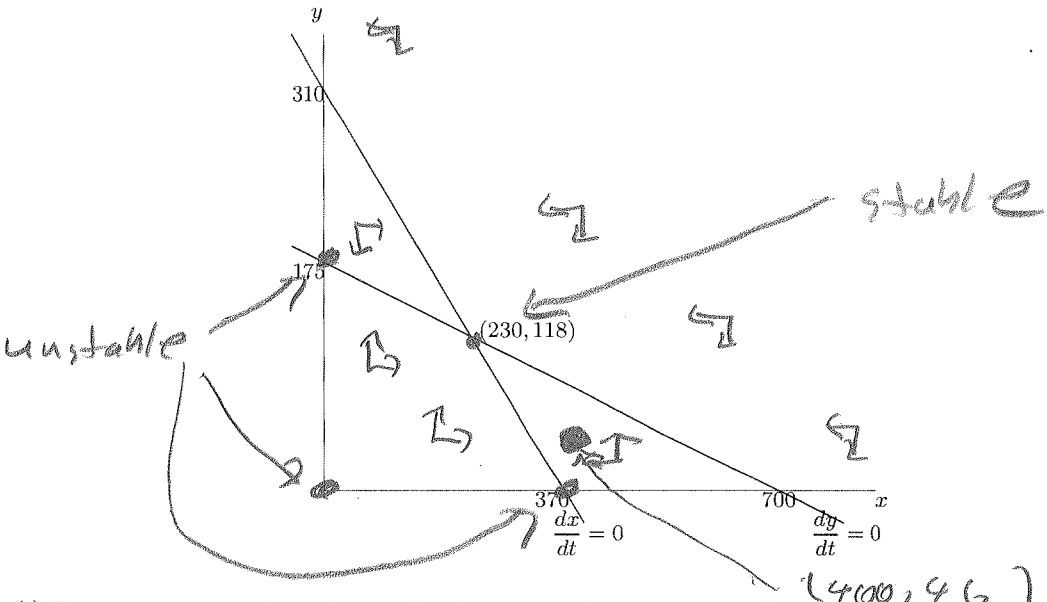
Mathematics 172 Test #3 Name: Key

You are to use your own calculator, no sharing.
Show your work to get credit.

1. (35 Points) The following is the system of rate equations for a two competing species (the x species and the y species)

$$\frac{dx}{dt} = r_1 x\left(\frac{K_1 - x - \alpha y}{K_1}\right) \qquad \frac{dy}{dt} = r_2 y\left(\frac{K_2 - y - \beta y}{K_2}\right)$$

(a) Here is one possible picture of the phase space for the system:



(i) Draw in arrows that shows the direction that points in the phase space are moving.

(ii) Use your answer to (b) to label all the equilibrium points as being either stable or unstable.

(iii) If $x(0) = 400$ and $y(0) = 40$ estimate $x(50)$ and $y(50)$.

$x(50) \approx$ 230 $\qquad y(50) \approx$ 118

(iv) If $x(0) = 0$ and $y(0) = 200$ estimate $x(75)$ and $y(75)$.

(stays on y axis)

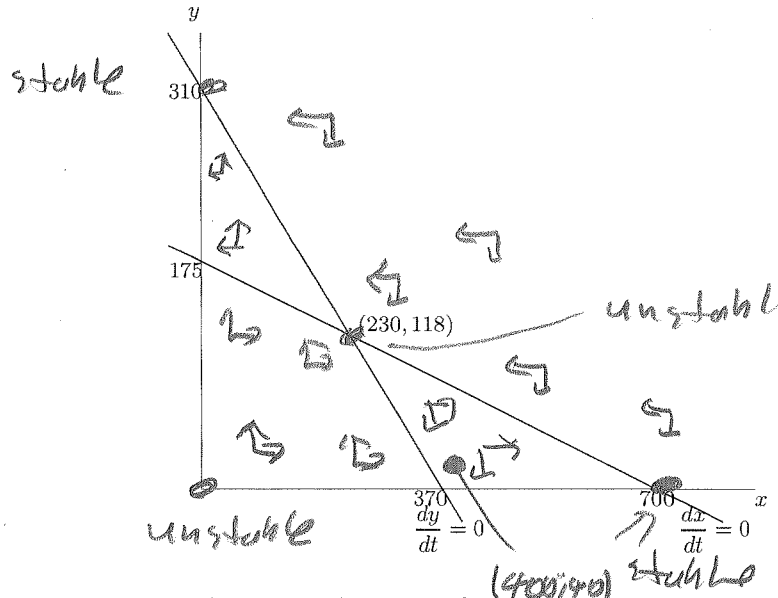
$x(75) \approx$ 0 $\qquad y(75) \approx$ 175

(v) Describe the long term behavior of the competition. That is competitive exclusion, competitive coexistence, species x dominates, or species y dominates. **Circle the appropriate one.**

(vi) In this case if there is a stable population of the x species is it possible for the y species to invade?

Yes or no. Yes

(b) Here is another possible picture of the phase space for the system:



(i) Draw in arrows that shows the direction that points in the phase space are moving.

(ii) Use your answer to (b) to label all the equilibrium points as being either stable or unstable.

(iii) If $x(0) = 400$ and $y(0) = 40$ estimate $x(75)$ and $y(75)$.

$x(75) \approx$ 700 $\quad$ $y(75) \approx$ 0

(iv) If $x(0) = 0$ and $y(0) = 200$ estimate $x(75)$ and $y(75)$.

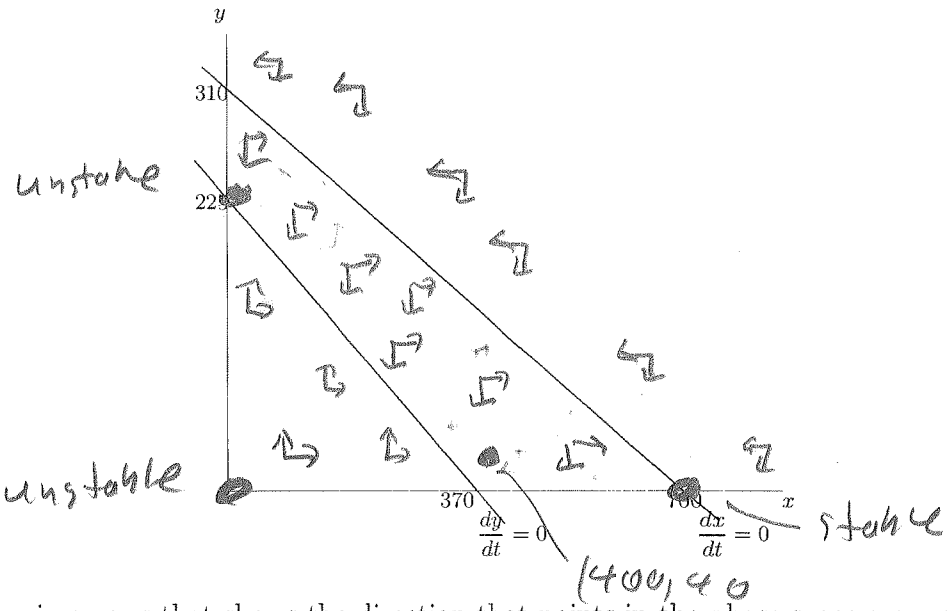
$x(75) \approx$ 0 $\quad$ $y(75) \approx$ 316

(v) Describe the long term behavior of the competition. That is competitive exclusion, competitive coexistence, species x dominates, or species y dominates. **Circle the appropriate one.**

(vi) In this case if there is a stable population of the x species is it possible for a small number of the y species to invade?

Yes or no. NO

(c) Here is yet another possible picture of the phase space for the system:



(i) Draw in arrows that shows the direction that points in the phase space are moving.

(ii) Use your answer to (b) to label all the equilibrium points as being either stable or unstable.

(iii) If $x(0) = 400$ and $y(0) = 40$ estimate $x(75)$ and $y(75)$.

$x(75) \approx$ 700 $\quad y(75) \approx$ 0

(iv) If $x(0) = 0$ and $y(0) = 200$ estimate $x(75)$ and $y(75)$.

$x(75) \approx$ 0 $\quad y(75) \approx$ 220

(v) Describe the long term behavior of the competition. That is competitive exclusion, competitive coexistence, species x dominates, or species y dominates. **Circle the appropriate one.**

(vi) In this case if there is a stable population of the x species is it possible for a small number the y species to invade?

Yes or no. NO

2. (15 Points) For the system describing two competing species

$$\frac{dx}{dt} = .15x\left(\frac{1{,}200 - x - .4y}{1{,}200}\right)$$
$$\frac{dy}{dt} = .4y\left(\frac{800 - y - .3x}{800}\right)$$

(a) What is the carrying capacity of the x species in the absence of the y species.

Carrying capacity is 1,200

(b) What is the intrinsic growth rate of the y species?

Intrinsic growth rate is .4

(c) Find all the equilibrium points of the system.

Equilibrium points are: (0,0), (0, 800), (1200, 0), (1000, 500)

We need to solve

$x(1200 - x - .4y) = 0$

$y(800 - y - .3x) = 0$

We easy see that (0,0), (0,800), (1200,0) are solutions. We then have to solve

$x + .4y = 1200$

$.3x + y = 800$

Y1 = (1200 − X)/.4

Y2 = 800 − .3X

Xmin = 0 Xmax = 1200

2nd calc 5: intersect

X = 1000

Y = 500 i.e. (1000, 500)

3. (25 Points) Consider a predator-victim system governed by the equations

$$\frac{dV}{dt} = .1V - .002VP = V(.1 - .002P)$$
$$\frac{dP}{dt} = -.2P + .0001VP = P(-.2 + .0001V)$$

(a) What is the intrinsic growth rate of the victim population?

Intrinsic growth rate is $.1$

(b) What is the intrinsic death rate of the predator population?

Intrinsic death rate is $.2$

(c) If $V(0) = 2{,}200$ and $P(0) = 55$ compute $V'(0)$ and $P'(0)$ and use this to give a sentence or two describing the initial behavior of the system.

$V'(0) =$ _____ $P'(0) =$ _____

$V'(0) = 2200(.1 - .002(55)) = -22$

$P'(0) = 55(-.2 + .0001(2200)) = 1.1$

V is decreasing at 22/ time unit

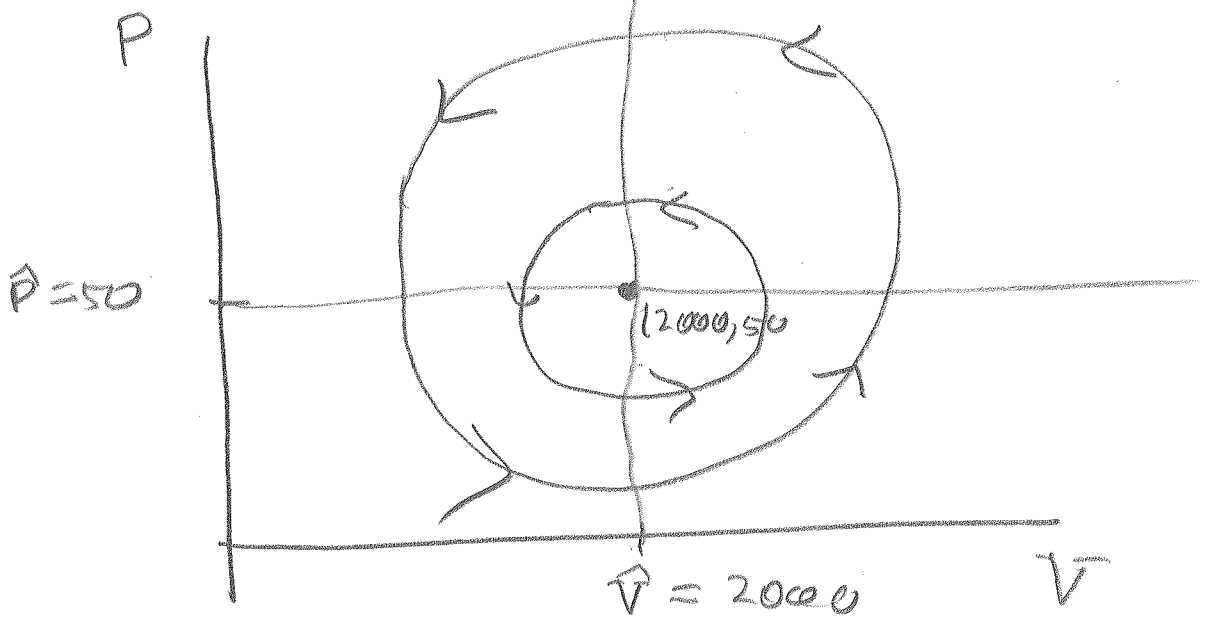
P is increasing at 1.1/ time unit

(d) What is the average number of victims and predators.

$\hat{V} =$ 2000 $\hat{P} =$ 50

$\hat{V} = \frac{.2}{.0001} = 2000$ $\hat{P} = 50$

(e) Draw the phase space including some curves with arrows showing how points move.



4. (25 Points) Islands off the cost of New Guinea are the homes of a species of fruit bat. The probability of an island populated by the bats having its population of bats going extinct is $p_e = .5$. The probability of an island with no bats being populated from New Guinea is $p_i = .3$.

(a) Let f be the fraction, or proportion, of the islands populated by the bats at a given time. Write a rate equation for f. (Note that a rate equation should include a $\frac{df}{dt}$ term and an equal sign.)

$\frac{dp}{dt} = .3(1-p) - .5p = .3 - .8p$

(b) Find the equilibrium point of the equation.

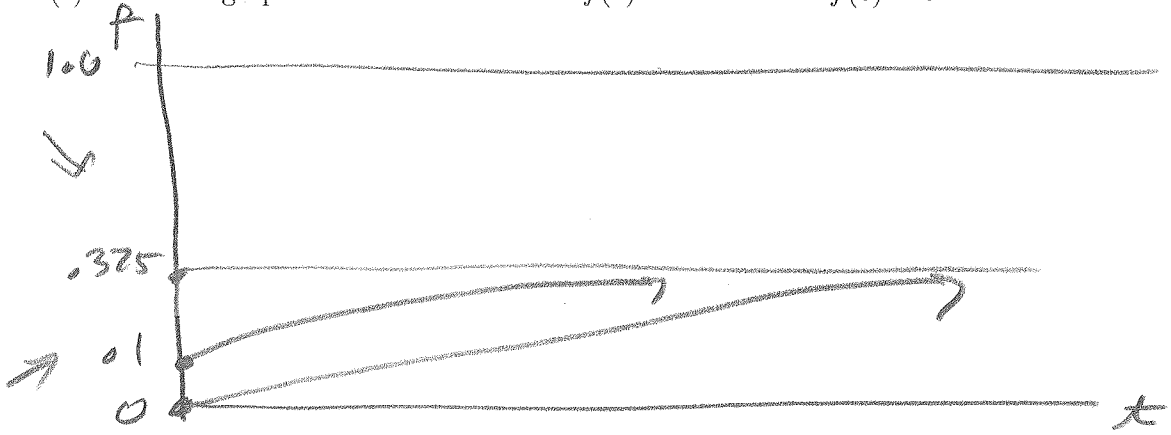
solve

$\frac{dp}{dt} = .3 - .8p = 0$

$p = \frac{.3}{.8} =$

Equilibrium point is .375

(c) Draw the graphs of the solutions with $f(0) = .1$ and with $f(0) = 0$.



(d) For the solution with $f(0) = .1$ estimate $f(100)$.

$f(100) \approx$ .375