Mathematics 172 Test #2 Name: Key

You are to use your own calculator, no sharing.

Show your work to get credit. **The problems are 20 points each.**

1. Define a discrete dynamical system by

$$N_{t+1} = N_t + .5N_t\left(1 - \frac{N_t^2}{90}\right)$$

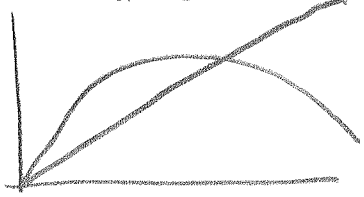
(a) If $N_0 = 5$ what are N_1 and N_2?

$N_1 = 5 + .5(5)\left(1 - \frac{5^2}{90}\right) = 6.806$ $N_1 =$ 6.806

$N_2 = 6.806 + .5(6.806)\left(1 - \frac{(6.806)^2}{90}\right)$ $N_2 =$ 8.456

(b) Plot $y = x + .5x\left(1 - \frac{x^2}{90}\right)$ and $y = x$ for $0 \le x \le 16$ on your calculator and make a rough sketch get here:

I used Xmin=0 Xmax=15



(c) What are the equilibrium points of this discrete dynamical system?

From picture N=0 is one Equilibrium points are: 0, 9.487

Solve $X + .5X\left(1 - \frac{X^2}{90}\right) = X$ on calculator to get $X = 9.487$

(d) Which of the equilibrium points are stable?

9.487

|slope| < 1

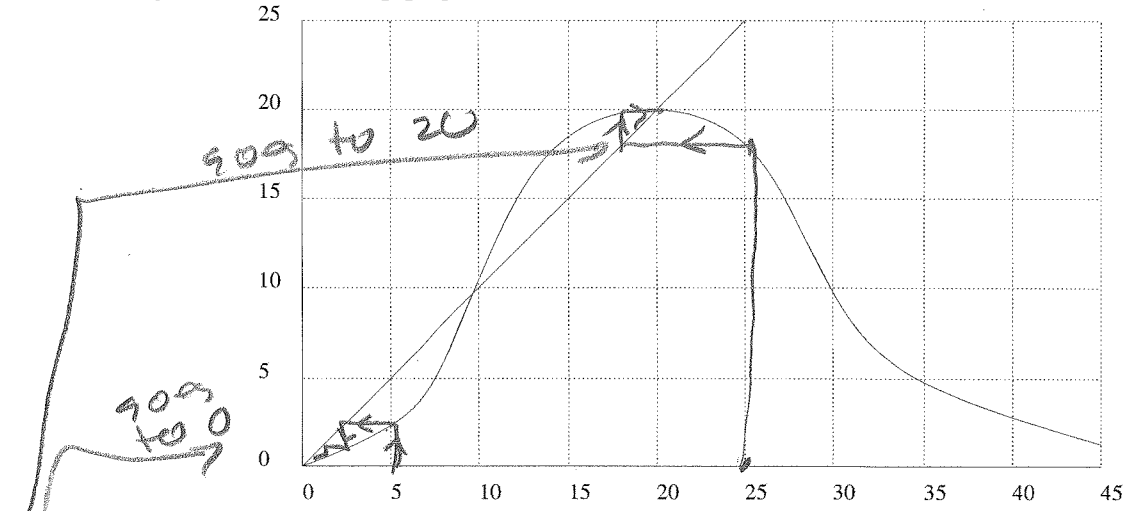
(e) Which of the equilibrium points are unstable?

0

2. A population of toads in a front yard grows by the rule

$$N_{t+1} = f(N_t)$$

where f has the following graph.



(a) What are the equilibrium points of this system?

Equilibrium points are 0, 9.9, 20

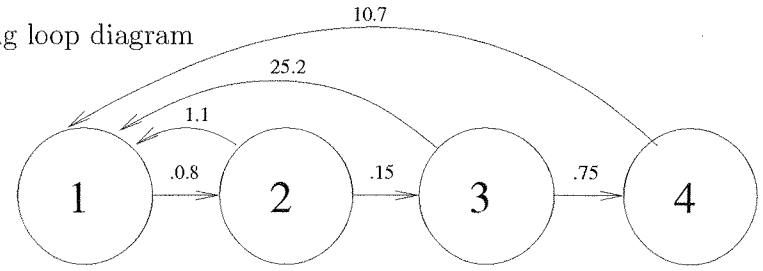
(b) Which of the equilibrium points are stable 0, 20

(c) Which of the equilibrium points are unstable 9.9

(d) If we start with 5 toads (that is $N_0 = 5$) estimate N_{50}. $N_{50} \approx$ 0

(e) If $N_0 = 25$ estimate N_{60}. $N_{60} \approx$ 20

3. For the following loop diagram



(a) What does the number 25.2 mean. (Your answer should be at least one complete sentence.)

stage 3 females have 25.2 female offspring that survive to stage 1 (on the average)

(b) What does the number .75 mean. (Your answer should be at least one complete sentence.)

75% of stage 3 females survive to stage 4.

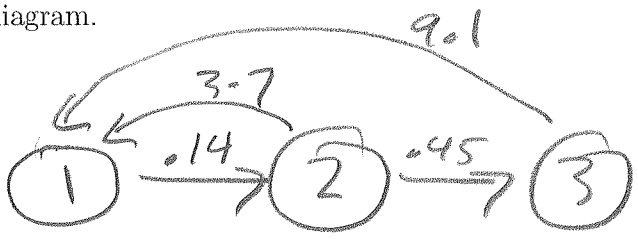
(c) What is the Leslie matrix:

$$\begin{bmatrix} 0 & 1.1 & 25.2 & 10.7 \\ .8 & 0 & 0 & 0 \\ 0 & .15 & 0 & 0 \\ 0 & 0 & .75 & 0 \end{bmatrix}$$

4. For the Leslie matrix

$$L = \begin{bmatrix} 0 & 3.7 & 9.1 \\ .14 & 0 & 0 \\ 0 & .45 & 0 \end{bmatrix}$$

(a) Draw the loop diagram.



(b) What does the number 9.1 mean?

stage 3 females have, on the average, 9.1 female offspring that survive to stage 1

(c) What does the number .45 mean?

45% of stage 2 females survive to stage 3

(d) If we start with 50 in stage 1, 7 in stage 2 and 3 in stage 3 then how many are in each stage after 35 years?

$\vec{n}(10) = \begin{bmatrix} 50 \\ 7 \\ 3 \end{bmatrix}$

$\vec{n}(35) = \begin{bmatrix} \\ \\ \end{bmatrix}$

Number in stage 1 171.2

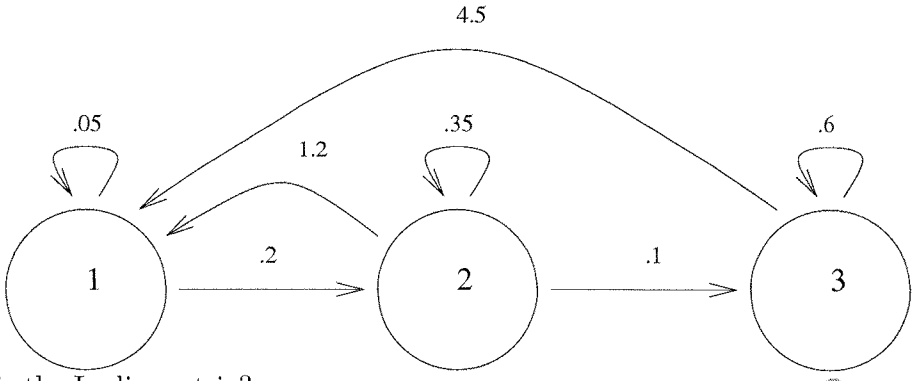
Number in stage 2 23.2

Number in stage 3 10.1

5. A naturalist taking a census of native Butterfly Weed in a small meadow. She can distinguish between three stages of the plant.

(1) Seedlings,
(2) Juveniles,
(3) Mature plants.

The life history is summarized by the following loop diagram:



(a) What is the Leslie matrix?

$$\begin{bmatrix} .05 & 1.2 & 4.5 \\ .2 & .35 & 0 \\ 0 & .1 & .6 \end{bmatrix}$$

(b) Starting with $\vec{n}(0) = \begin{bmatrix} 20 \\ 7 \\ 3 \end{bmatrix}$ compute $\vec{n}(30)$ and $\vec{n}(31)$ and use these to find the per capita growth rate r. (Recall that $\lambda = 1+r$ and be sure to use at least 4 decimal places in your calculations.)

$$\vec{n}(30) = \begin{bmatrix} 2.7311 \\ .9401 \\ .2840 \end{bmatrix} \qquad \vec{n}(31) = \begin{bmatrix} 2.5427 \\ .8753 \\ .2644 \end{bmatrix}$$

We got 3 answers for λ

$\lambda = \frac{2.5427}{2.7311} = .9310$

$\lambda = \frac{.8753}{.9401} = .9311$

$\lambda = \frac{.2644}{.2840} = .9310$

so to 3 dec. places $\lambda = .931$

thus $r = \lambda - 1 = -.069$

(c) What is the stable age distribution?

From (b) we see that at $t=30$ we have pretty much reached the stable age distribution.

Percent that are seedlings: 69.0%

Percent that are juveniles: 23.8%

Percent that are mature: 7.2%

Total $= 2.7311 + .9401 + .2840 = 3.9552$

seedlings $= \frac{2.7311}{3.9552} = 69.0\%$

Juveniles $= 23.8\%$

mature $= \frac{.2840}{3.9525} = 7.2\%$