**Mathematics 172 Test #1** Name: Key

**You are to use your own calculator, no sharing.**
**Show your work to get credit.**

**1.** (10 points) The weight, $W$, in pounds of a mako shark shark is proportional the cube of its length, $L$, in feet. A 2 foot mako shark weights 2.25 pounds.

(a) Give a formula for $W$ in terms of $L$. $W =$ $.28125 L^3$ lbs

We are given that $W = cL^3$ for some constant $c$. when $L = 2$, $W = 2.25$ → so $2.25 = c(2)^3$ Thus $c = \frac{2.25}{2^3} = .28125$

(b) Estimate the weight of a 9 foot mako shark. The weight is 205.03 lbs

If $L = 9$, $W = .28125(9)^3$
$= 205.03$

(c) Estimate the length of a 90 pound shark. The length is 6.84 ft

If $W = 90$ we have
$90 = .28125 L^3$
$L^3 = \frac{90}{.28125}$ → $L = \left(\frac{90}{.28125}\right)^{\frac{1}{3}} = 6.840$

**2.** (10 points)A agriculturist plans to raise tilapia in tanks. She finds that if she doubles the tank volume, $V$, in gallons, the weight, $W$, in pounds, of the fish she harvests is doubled. If she triples the tank volume, the weight of the harvest is tripled.

(a) If 100 gallon tank gives a harvest of 12 pounds of fish then give a formula for for $W$ in terms of $V$. $W =$ $.12V$ lbs

The hypothesis imply
$W = cV$ for a constant $c$
when $V = 100$, $W = 12$ which leads to
$12 = c(100)$ $c = \frac{12}{100} = .12$

(b) If she wishes to harvest 50 pounds of fish, how large of a tank should she use?

Volume of tank is 416.67 lbs

When $W = 50$ we have
$50 = .12V$
$V = \frac{50}{.12} = 416.67$

**3.** (15 points) Eight tigers are released in a national park in India and the population has discrete exponential growth with a per capita rate of $r = .6$ (tigers/year)/tiger.

(a) Write a formula for $N_t$, the number of tigers after $t$ years.

This is ~~discrete~~ exp. growth so we have

$N_t = N_0 \lambda^t$

where $N_0 = 8$, $\lambda = 1 + r = 1.6$

$N_t =$ $8(1.6)^t$

(b) How many tigers are there after five years?

$N_5 = 8(1.6)^5$
$= 83.89$

Number of tigers after five years = 83.89

(c) How long does it take the population of tigers to reach 500?

We wish to solve

$N_t = 8(1.6)^t = 500$

$(1.6)^t = \frac{500}{8}$

$t \ln(1.6) = \ln(500/8)$

$t = \frac{\ln(500/8)}{\ln(1.6)} = 8.798$

Time to reach 500 8.80

**4.** (15 points)A species of goose has an annual birth rate of $b = 1.4$ geese/goose and an annual death rate of $d = .8$ geese/goose.

(a) What is the discrete growth factor $r$?

$r = b - d = 1.4 - .8 = .6$

$r =$ $.6$

(b) What is the discrete finite rate of increase $\lambda$?

$\lambda = 1 + r = 1.6$

$\lambda =$ $1.6$

(c) If a flock starts with 15 geese, give a formula for the number, $N_t$, of geese $t$ years later.

$N_t = N_0 \lambda^t = 15(1.6)^t$

$N_t =$ $15(1.6)^t$

(d) How many geese are there after 25 years?

$N_{25} = 15(1.6)^{25} = 1,901,476.$

$N_{25} =$ 1,901,476.

**5.** (10 points) Let $N(t)$ be the weight in pounds of algae in a pond after $t$ weeks. Assume that the algae has unrestricted continuous growth. That is it satisfies the rate equation

$$\frac{dN}{dt} = rN$$

where the intrinsic growth rate, $r$, is constant. We start with 1.2 pounds of algae in the pond and after 3 weeks there is 3.8 pounds.

(a) Give a formula for $N(t)$. $N(t) =$ $1.2e^{.3842t}$

$N(t) = N_0 e^{rt} = 1.2e^{rt}$

to find $r$: $N(3) = 1.2e^{r(3)} = 3.8$

$e^{3r} = \frac{3.8}{1.2}$

$3r = \ln\left(\frac{3.8}{1.2}\right)$

$r = \frac{\ln\left(\frac{3.8}{1.2}\right)}{3} = .3842$

(b) How long until there is 1,000 pounds of algae in the pond?

Solve

$N(t) = 1.2e^{.3842t} = 1000$

Time to get 1,000 pounds is 17.50 weeks

$e^{.3842t} = \frac{1000}{1.2}$

$.3842t = \ln(1000/1.2)$

$t = \frac{\ln(1000/1.2)}{.3842} = 17.50$

**6.** (15 points) Let $N(t)$ satisfy the rate equation

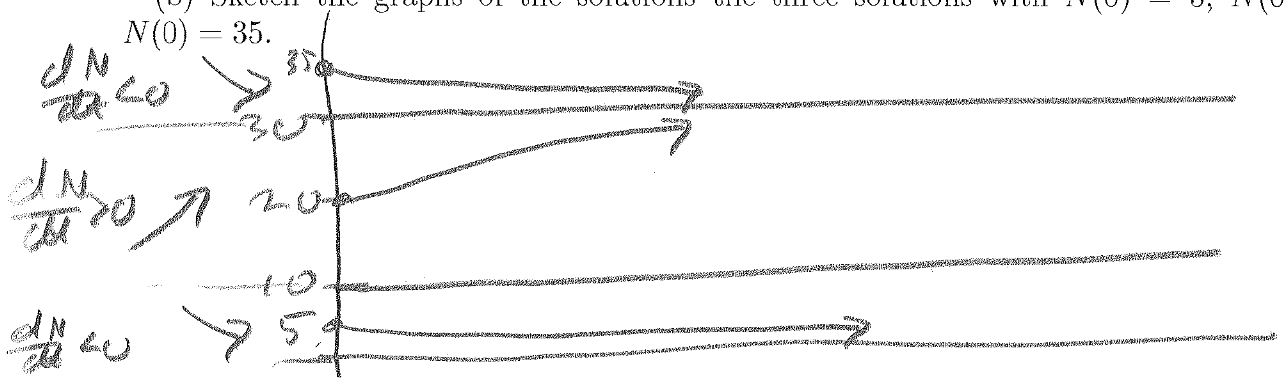
$$\frac{dN}{dt} = -.3N(N-10)(N-30)$$

(a) What are the equilibrium points of this equation?

Solve $\frac{dN}{dt} = -.3N(N-10)(N-30) = 0$ to get $N = 0, 10, 30$

$N=0,\ N=10,\ N=30$

(b) Sketch the graphs of the solutions the three solutions with $N(0) = 5$, $N(0) = 20$, and $N(0) = 35$.




(c) If $N(0) = 23$ estimate $N(100)$. $N(100) \approx$ 30

**7.** (15 points) Twenty pounds of duck weed is introduced into a pond. If $N(t)$ is the weight of the duck weed $t$ days after it is introduced to the pond, then $N(t)$ grows logistically with an intrinsic growth rate of $r = .6$ (lbs/lb)/day and a carrying capacity of 75 pounds.

(a) Write down the corresponding logistic equation.

$\frac{dN}{dt} = rN(1-\frac{N}{K})$, i.e. $\frac{dN}{dt} = .6N(1-\frac{N}{75})$

(b) What is the initial growth rate? Initial growth rate is $8.8$ lbs/day

$N(0) = 20$ so

$N'(0) = .6(20)(1-\frac{20}{75}) = 8.8$

(c) What is the rate of change when $N = 50$ lbs? $\frac{dN}{dt} =$ $10$ lbs/day

$\left.\frac{dN}{dt}\right|_{N=50} = .6(50)(1-\frac{50}{75}) = 10$

(d) Estimate the amount of duck weed in the pond the day after the day when $N = 50$.

We have 50 lbs and it is increasing at 10 lbs/day so on the next day there is $\approx 50+10 = 60$

The estimate is 60 lbs.

(e) Estimate the amount of duck week in the pond after 200 days.

By 200 days it will have reached the carrying capacity.

$N(200) \approx$ $75$

**8.** (10 points) Mosquitofish are being raised in a polluted pond such that the intrinsic rate of growth is $r = -0.15$ (fish/fish)/month. Let $N(t)$ be the number of fish in the pond after $t$ months. To keep the fish from dieing out the pond is stocked are a rare of 100 fish/month.

(a) Write the rate equation satisfied by $N$ if there is no stocking.

$\frac{dN}{dt} = -.15N$

(b) Write the rate equation satisfied by $N$ if there there stocking at a continuous rate of 100 fish/month.

$\frac{dN}{dt} =$ $-.15N+100$

(c) With stocking what is the stable population size?

solve $\frac{dN}{dt} = -.15N+100 = 0$ Stable population size is $666.67$

$-.15N = -100$

$N = \frac{100}{.15} = 666.67$