Mathematics 172

Quiz #5 Name: Key

You must show your work to get full credit.

Part of the point of this quiz is to show how fast exponential functions grow.

Under ideal conditions the bacterium E. coli will double every 1/3 hour. A single E. coli weighs 10^{-15}kg.

1. We start a colony E. coli with a single E. coli. Assuming that the weight of the colony doubles every 1/3 hour, give a formula for the weight, $W(t)$, of the colony after t hours.

$W(t) = W(0)\lambda^t$

$= 10^{-15}\lambda^t$

$W(1/3) = 10^{-15}\lambda^{\frac{1}{3}} = 2W(0) = 2\cdot 10^{-15}$

$\lambda^{\frac{1}{3}} = \frac{2\cdot 10^{-15}}{10^{-15}} = 2$

$\lambda = (\lambda^{\frac{1}{3}})^3 = 2^3 = 8$

$W(t) = 10^{-15}(8)^t$ kg

2. The weight of the Earth is 6.0×10^{24}kg. How long does it take for our colony of E. coli to have the same weight as the Earth?

Time to reach mass of Earth is 44.04 hours

That is we wish to find t such that

$W(t) = 10^{-15}(8)^t = 6\times 10^{24}$

$8^t = \frac{6\times 10^{24}}{10^{-15}} = 6\times 10^{24+15} = 6\times 10^{39}$

$t\ln(8) = \ln(6\times 10^{39})$

$t = \frac{\ln(6\times 10^{39})}{\ln(8)} = 44.04$