Mathematics 172

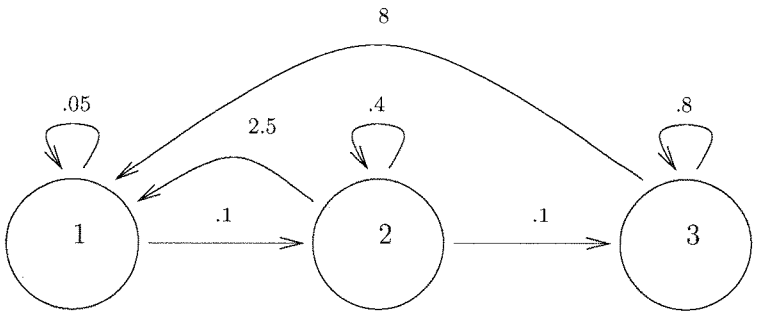
Quiz #34 Name: Key

You must show your work to get full credit.

1. A naturalist taking a census of native rhododendrons (a plant relates to azaleas) can distinguish between three stages of the plant.

(1) Seedlings,
(2) Juveniles,
(3) Mature plants.

The life history is summarized by the following loop diagram.



(a) What is the average number of seedlings per year produced by a juvenile.

Average number is 2.5

(b) What is the Leslie matrix?

$$\begin{bmatrix} .05 & 2.5 & 8 \\ .1 & .4 & 0 \\ 0 & .1 & .8 \end{bmatrix}$$

Starting with $\vec{n}(0) = \begin{bmatrix} 10 \\ 2 \\ 1 \end{bmatrix}$ compute $\vec{n}(30)$ and $\vec{n}(31)$ and use these to find the per capita growth rate r. (Be sure to use at least 4 decimal places in your calculations.)

From calculator:

$\vec{n}(30) = \begin{bmatrix} 25.7911 \\ 4.1345 \\ 1.8474 \end{bmatrix}$

$\vec{n}(31) = \begin{bmatrix} 26.4050 \\ 4.2329 \\ 1.8914 \end{bmatrix}$

We want to find $\lambda = (1+r)$ such that

$r =$ .024

$\vec{n}(31) = \lambda \vec{n}(30)$. This leads to 3 equations for λ

$\lambda = \frac{26.4050}{25.7911} = 1.024$

$\lambda = \frac{4.2329}{4.1345} = 1.024$

$\lambda = \frac{1.8914}{1.8474} = 1.024$

Thus $r = \lambda - 1 = .024$

2. A population of yeast is growing logistically with an intrinsic growth rate of $r = .8$ grams/day and a carrying capacity of 100 grams. Let $A(t)$ be the number of grams of yeast after t days.

(a) Write the rate equation for A.

$$\frac{dA}{dt} = .8A\left(1 - \frac{A}{100}\right)$$

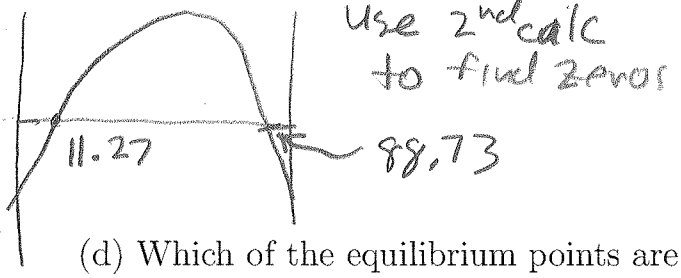
(b) A baker starts using the yeast at a constant rate of 8 grams/day. What is the new rate equation satisfied by A?

$$\frac{dA}{dt} = .8A\left(1 - \frac{A}{100}\right) - 8$$

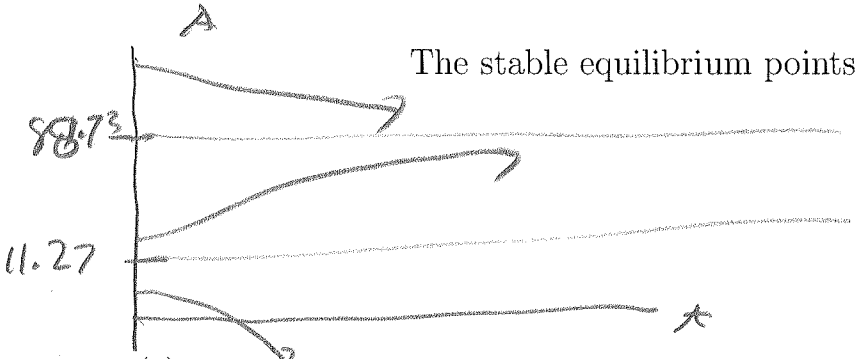
(c) What are the equilibrium points for the equation of Problem (b)?

Graph
$Y1 = .8X(1 - X/100) - 8$

The equilibrium points are 11.27, 88.73



(d) Which of the equilibrium points are stable?



The stable equilibrium points are 88.73

(e) What is the greatest rate that the baker can harvest the yeast with out killing off the population of yeast?

Going back to the equation of a graph
$Y1 = .8X(1 - X/100)$
and use 2nd calc to find the maximum value

Maximal rate is: 20