

Mathematics 172

Quiz #30

Name: Key

You must show your work to get full credit.

Here is how to a rate equation of the form

$$\frac{dP}{dt} = kP + c$$

where k and c are constants. To be specific let us use constants:

$$\frac{dP}{dt} = -2P + 100.$$

and assume $P(0) = 40$.

1. Find the equilibrium point. (*Hint:* The answer is 50, but you should show your work.)

$$\begin{aligned} \frac{dP}{dt} &= -2P + 100 = 0 \\ -2P &= -100 \\ P &= \frac{100}{2} = 50 \end{aligned}$$

Equilibrium point is 50

2. Let $y = P - 50$ be the difference P and its equilibrium value. Then $P = y + 50$. Find the rate equation for y .

$$\begin{aligned} P &= y + 50 \\ \frac{dP}{dt} &= \frac{dy}{dt} \end{aligned}$$

Rate equation is $\frac{dy}{dt} = -2y$

so
$$\frac{dy}{dt} = \frac{dP}{dt} = -2P + 100 = -2(y + 50) + 100 = -2y$$

3. Find $y(0) = P(0) - 50 = 40 - 50 = -10$ $y(0) = \underline{-10}$

4. Find a formula for $y(t)$.

$$\begin{aligned} y(t) &= y(0) e^{-2t} \\ &= -10 e^{-2t} \end{aligned}$$

$y(t) = \underline{-10 e^{-2t}}$

5. Finish by finding a formula for $P(t)$

$$\begin{aligned} P(t) &= \underline{50 - 10 e^{-2t}} \\ P &= y + 50 = -10 e^{-2t} + 50 \end{aligned}$$